Electric Potential of Conductors

When reviewing Gauss's law, we concluded that if you place a conductor in an electric field the resulting electric field anywhere inside the conductor is zero. We also used Gauss's law to show that if we place an excess charge on a conductor the charge will remain on the surface only. This was true even for a conductor with an empty cavity inside. We formally stated these results as shown below.

Theorem for Static Charges and Conductors
If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor. In other words, the electric field inside of a conductor is zero in the static situation.
Conducting Shells
A conducting shell with an excess charge will still have this excess charge move to the outside of the shell and will contain zero electric field inside the shell.

In this section we investigate the electric potential inside a conductor. To start let's write the definition for the change in the potential.

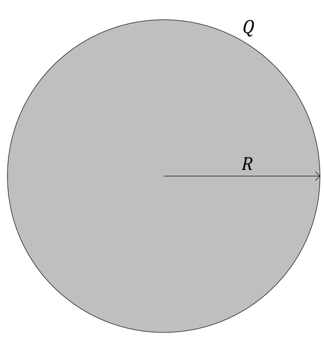
$$\Delta V = -\int_i^f \boldsymbol{E} \cdot \boldsymbol{dl}$$

And if the electric field is zero, as it is inside a conductor, we have the following:

$$V_f - V_i = -\int_i^f \boldsymbol{0} \cdot \boldsymbol{dl}$$
$$V_f = V_i$$

Which shows that the *electric potential inside a conductor is constant*. To understand how we can determine this constant value we will look at an example for a spherical conductor. Note that as the example shows a solid sphere, the same results will hold for a spherical shell since the electric field inside the shell is also zero.

The figure below shows a solid conducting sphere of radius R with a uniformly distributed surface charge of Q. The magnitude of the electric field inside and outside the sphere can be found using Gauss's law, which we reviewed in a previous section.



$$E = \begin{cases} 0, & r < R \\ k\dfrac{Q}{r^2}, & r \geq R \end{cases}$$

To find the potential at a distance r from the center of the sphere we can start by finding the potential for $r \geq R$.

$$\Delta V = -\int_{r}^{\infty} E dr$$

$$\Delta V = -kQ\int_{r}^{\infty} \frac{1}{r^2} dr$$

$$\Delta V = kQ\left[\frac{1}{\infty} - \frac{1}{r}\right]$$

$$V(\infty) - V(r) = -k\frac{Q}{r}$$

Letting $V(\infty) = 0$, we have

$$V(r) = k\frac{Q}{r}, \qquad r \geq R$$

For $r < R$ we find the following

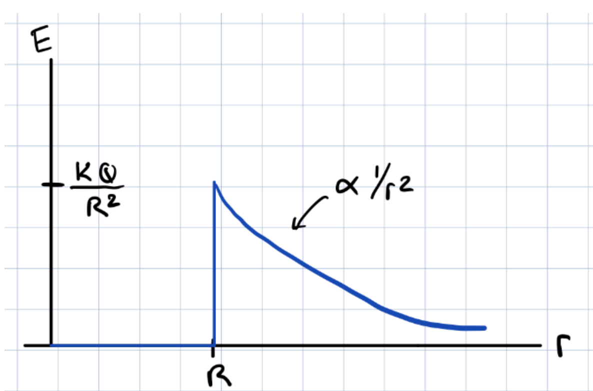
$$\Delta V = -\int_{0}^{r} E dr$$

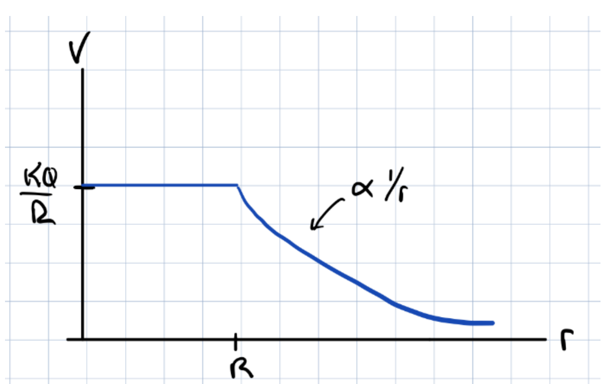
$$V(r) - V(0) = -\int_{0}^{r} 0 dr$$

$$V(r) = V(0)$$

Which, as expected, shows that the potential within a conductor is constant. The value will be equal to the value at the surface, i.e., $k\frac{Q}{R}$. Therefore, for a solid spherical conductor we have

$$E = \begin{cases} 0, & r < R \\ k\frac{Q}{r^2}, & r \geq R \end{cases} \qquad\qquad V = \begin{cases} k\frac{Q}{R}, & r < R \\ k\frac{Q}{r}, & r \geq R \end{cases}$$

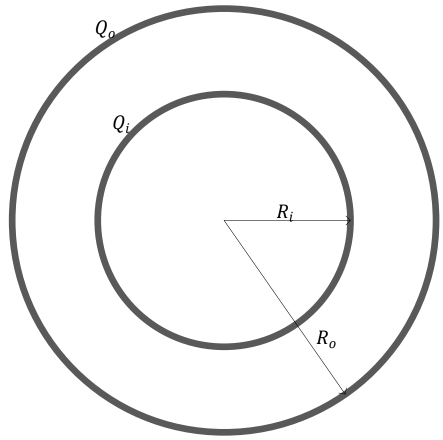




Note: This result holds for spherical shells also.

Following this integral procedure for more complex examples can become time consuming. However, we can directly apply these results to more complex scenarios by using the superposition principle. Recall that when applying the principle of superposition, we sum the potentials from each charge *individually*. The following example illustrates this procedure.

Find the electric potential at a distance r from the center of two thin conducting concentric spherical shells shown below. The interior shell has a charge of Q_i and radius R_i, while the outer shell has a charge of Q_o and radius of R_o.



1. *Inside the inner shell,* $r < R_i$:

Using the principle of superposition, we can write:

$$V = V_{Q_i} + V_{Q_o}$$

Where, V_{Q_i} is the potential from the charge on the inner shell, and V_{Q_o} is the potential from the charge on the outer shell. From the first example we know that the potential inside a spherical shell is equal to the potential on the surface of the shell, therefore we have the following:

$$V = k\left(\frac{Q_i}{R_i} + \frac{Q_o}{R_o}\right)$$

2. *Between the two shells,* $R_i < r < R_o$:

We start with the same equation.

$$V = V_{Q_i} + V_{Q_o}$$

However, in this case since we are now *outside* the inner shell, the potential due to Q_i is not constant, but is a function of the distance.

$$V = k\left(\frac{Q_i}{r} + \frac{Q_o}{R_o}\right)$$

3. *Outside both shells,* $r > R_o$*:*

In this case we are outside both shells, therefore the potential from both shells are functions of the distance.

$$V = V_{Q_i} + V_{Q_o}$$
$$V = k\left(\frac{Q_i}{r} + \frac{Q_o}{r}\right)$$
$$V = k\left(\frac{Q_i + Q_o}{r}\right)$$

Final Summary for Electric Potential

Electrical Potential of Charged Conductors
The electric potential inside a charged conductor is constant (i.e. all points within a charged conductor form an equipotential space.) The electric potential at points on the interior of a charged spherical conductor, (solid or shell), are equal to the electric potential at the surface.

By: ferrantetutoring