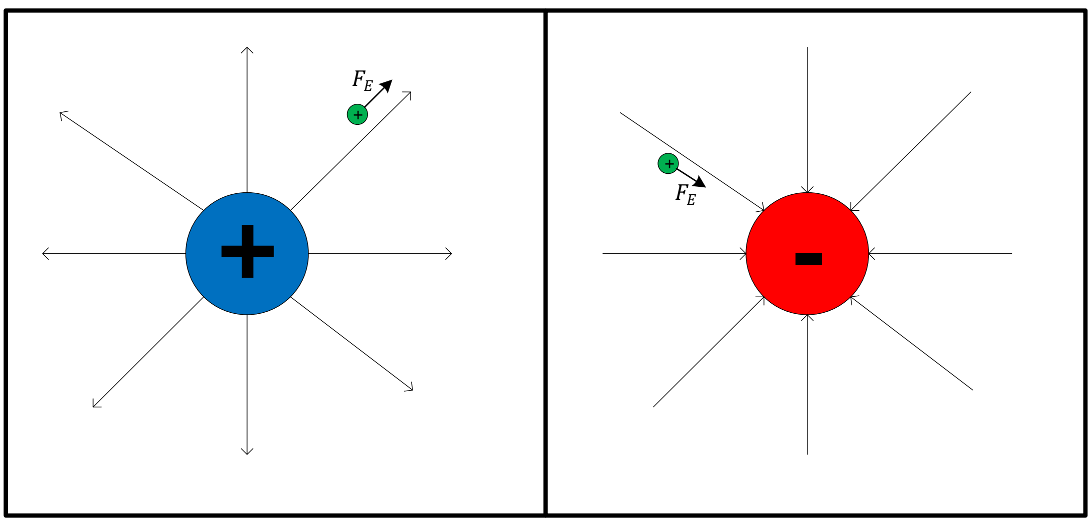
Physics 2 E&M - Electric Field

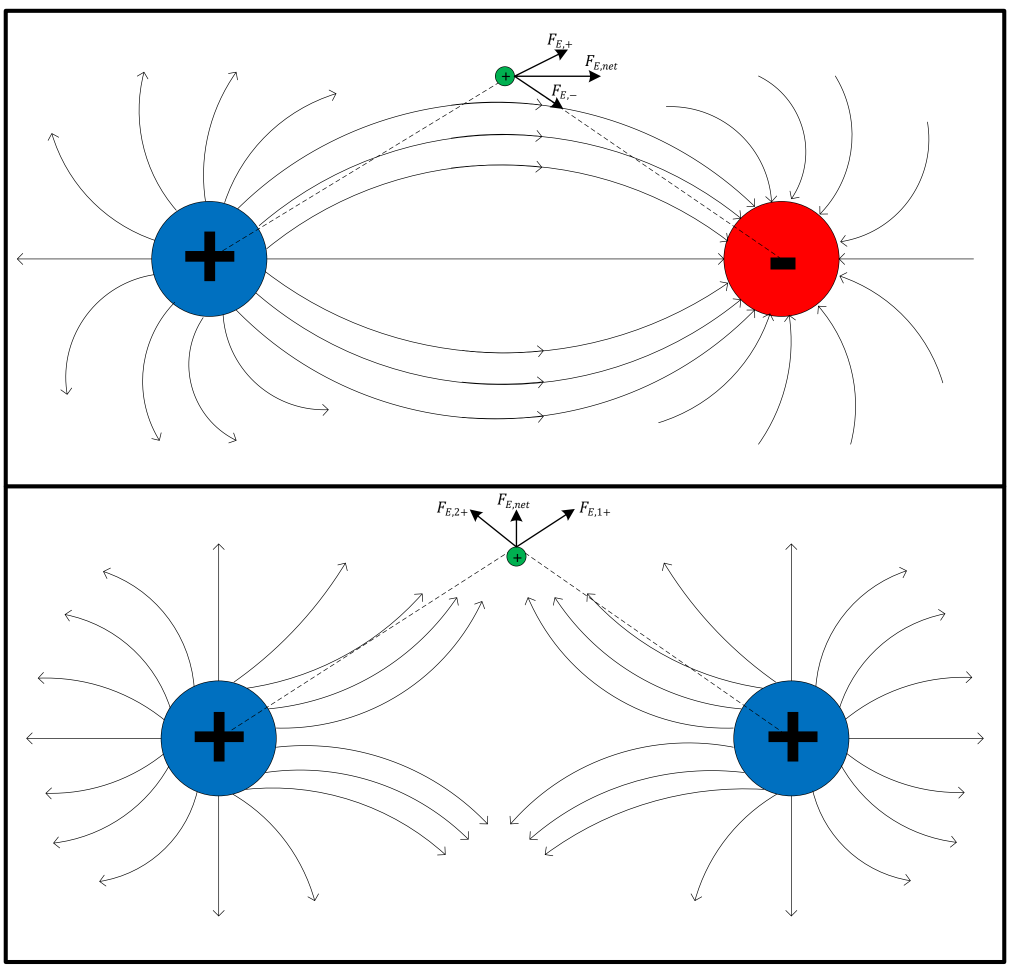
Let's say we wanted to move a cup of coffee across a table. We would apply a force, a "push" or a "pull", depending on the direction we wanted to move the cup. In either case we would need to make *physical contact* with the cup to apply this force. However, what we have seen in the previous lesson is that two charged particles apply forces on each other *without making physical contact*. This is also the case for the gravitational force, and even Newton felt uncomfortable with this idea of a force acting without direct contact. In wasn't until later that Michael Faraday, working with the electrical force, developed a way to deal with this idea. He imagined an *electric field* that extends from a charged object and permeates all space. When a second charge is placed in this field, it will "feel" the force from the first charge. Recall that electrical forces can be either attractive or repulsive and by convention we define the direction of the electric field according to how it would apply the force to a *positive test charge*. Below is an illustration of the electric field for a positively and negatively charged object.



For the positively charged object the electric field lines are directed away from the object since that is the direction in which a positive test charge, shown in green, would feel the electric force, F_E. On the other hand, for the negatively charged object the electric field lines are directed inward. Again, this is because the positive test charge would feel a force in this direction. In general, when placing a positive test charge in an electric field, the force applied to the test charge will be in a direction that is tangent to the field line. When there is a single isolated charge, the tangent line is trivial since all field lines are linear.

What happens when we have more than one charge? In the previous lesson we saw that the net electrical force from multiple charges is the vector sum of the forces from each charge. Fortunately, the same is true for the electric field. Each charge establishes an electric field which can be described at each point is space with a magnitude and a direction, i.e., *an electric field is referred to as type of vector field*. When there are multiple charges in the same vicinity the electric field at any point in space is the vector sum of the vector field values from each charge. In this way electric field lines can be made to curve. The figure below shows two different scenarios.

- *One Positive and One Negative Charge*
  - The field lines emerge from the positive charge as they do for an isolated positive charge, but in this case, they wrap back around and terminate on the negative charge. As shown, if we put a test charge in the middle of the two charges and add the force vectors on this charge, we get a net force vector that is tangent to the electric field line at that point in space.
- *Two Positive Charges*
  - The field lines emerge from both positive charges, but they bend away from each other when they meet between the two charges. Again, when we place a test charge in the middle of the two charges, we find the net force vector is, as expected, tangent to the field line at that point in space.



Given the above description we can list three rules for drawing electrical field lines as follows.

Rules for drawing Electric Field Lines
1. Field lines begin and end at electric charges (or at infinity). 2. At any point in space the net electric force vector is tangent to the electric field line at that point and it points in the same direction. 3. Field lines can never intersect each other. (Otherwise at the point of intersection we could draw two different tangent lines indicating two different directions for the net force, which is not physically possible.)

The above description was qualitative only. Let's now discuss how the electric field is described quantitatively using mathematics. The electric force on charge 1 by charge 2 is given as follows.

$$\boldsymbol{F}_{E,12} = \frac{kq_1q_2}{r^2}\hat{\boldsymbol{r}}_{12}\ N$$

Where, $\hat{\boldsymbol{r}}$ is a unit vector that points in the direction of the force.

Therefore, to compute the force, both the "acting" charge, q_1, and "receiving" charge, q_2, need to be considered. On the other hand, we would like the electric field to give us knowledge of what force the "receiving" charge would ***if*** we were to place it near the "acting" charge. In other words, the electric field should be a property of a single charge alone. One way to accomplish this is to simply remove the q_2 term from the force equation. The results is what we refer to as the electric field. The electric field at a point, p, that results from a point charge, q_1, is then given as shown.

$$\boldsymbol{E}_p = \frac{kq_1}{r^2}\hat{\boldsymbol{r}}_{1p}\ \ N/C$$

With this expression we can now place a charge, q, at a given location, p, in the vicinity of an electric field, and find the force that acts on that charge as follows.

$$\boldsymbol{F}_{E,p} = \boldsymbol{E}_p q$$

The relationship between the electric force and the electric field can then be written as follows.

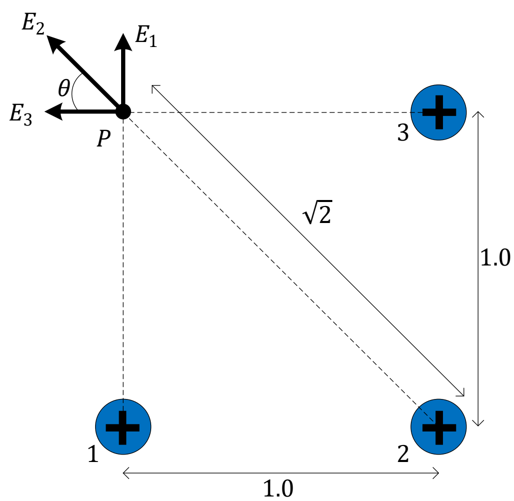
$$\boldsymbol{F}_E = \boldsymbol{E}q \ \rightarrow \boldsymbol{E} = \frac{\boldsymbol{F}_E}{q}$$

Furthermore, as mentioned above, the electric field at a point p from multiple point charges is computed as the vector sum of the electric field from each charge.

$$\boldsymbol{E}_p = \sum_{i=1}^{N} k\frac{q_i}{(r_{ip})^2}\hat{\boldsymbol{r}}_{ip}$$

Example 1: Three equally charged particles of $Q = 3.25\ uC$ are arranged in three corners of a square with side length equal to $1.0\ m$. Find the electric field at the fourth corner.

Solution 1: We start by drawing the charges at three corners of a square as indicated. We also mark the point P at the fourth corner of the square and show the direction of the E field vectors resulting from each charge.



According to what we learned above we can write the net electric field as the vector sum of the field vectors from each of the three charges.

$$\boldsymbol{E}_p = \sum_{i=1}^{3} \boldsymbol{E}_i$$

Let's first express the electric field from each charge in vector form, starting with $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$.

$$\boldsymbol{E}_1 = \langle 0, k\frac{Q_1}{(r_{1p})^2}\rangle = kQ\langle 0, 1\rangle \qquad \boldsymbol{E}_3 = \langle - k\frac{Q_3}{(r_{3p})^2}, 0\rangle = kQ\langle -1,0\rangle$$

Where, we used the fact that $r_{1p} = r_{3p} = 1$.

The electric field from the second charge has both an x and y component.

$$\boldsymbol{E}_2 = \langle - k\frac{Q_3}{(r_{3p})^2}cos(\theta), k\frac{Q_3}{(r_{3p})^2}sin(\theta)\rangle$$

$$\boldsymbol{E}_2 = kQ\,\langle -\frac{1}{(\sqrt{2})^2}cos(45^\circ), \frac{1}{(\sqrt{2})^2}sin(45^\circ)\rangle$$

$$\boldsymbol{E}_2 = kQ\,\langle -\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\rangle$$

The net electric field is then computed below.

$$\boldsymbol{E}_p = \sum_{i=1}^{3} \boldsymbol{E}_i$$

$$\boldsymbol{E}_p = kQ\langle 0, 1\rangle + kQ\left\langle -\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right\rangle + kQ\langle -1,0\rangle$$

$$\boldsymbol{E}_p = kQ\left\langle -\left(1+\frac{\sqrt{2}}{4}\right), \left(1+\frac{\sqrt{2}}{4}\right)\right\rangle$$

$$\boldsymbol{E}_p = kQ\left(1+\frac{\sqrt{2}}{4}\right)\langle -1,1\rangle$$

$$\boldsymbol{E}_p = 9E^9 \cdot 3.25\,E^{-6}\left(1+\frac{\sqrt{2}}{4}\right)\langle -1,1\rangle$$

$$\boldsymbol{E}_p = \langle -39591, 39591\rangle\, N/C$$

The magnitude of the electric field is given as follows.

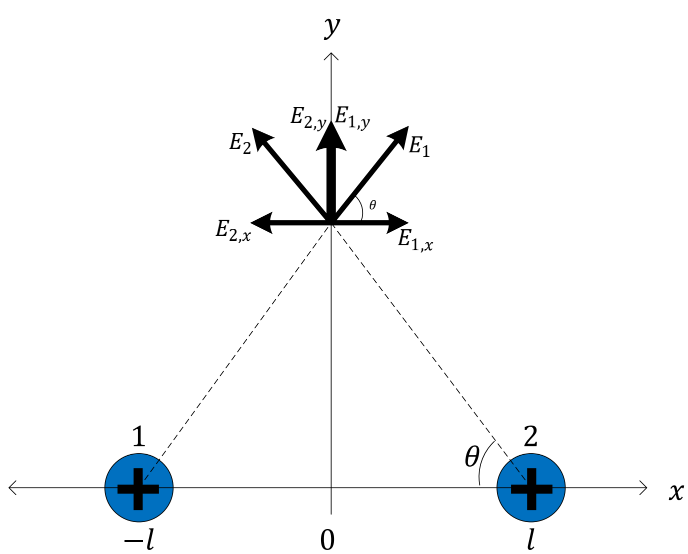
$$|\boldsymbol{E}_p| = \sqrt{(-39591)^2 + (39591)^2}$$
$$|\boldsymbol{E}_p| = \sqrt{2} \cdot 39591$$
$$|\boldsymbol{E}_p| = 55991\, N/C$$

Finally, since the x and y components are equal in magnitude, $\varphi = 135^{\circ}$ measured from the positive x-axis.

Example 2: Two equal positive charges are placed along the x axis at $-l$ and $+l$.

a. Determine the E field at any point along the y axis.
b. Find the point on the y axis where the E field is maximum.

Solution 2a: We start by drawing the scenario below.



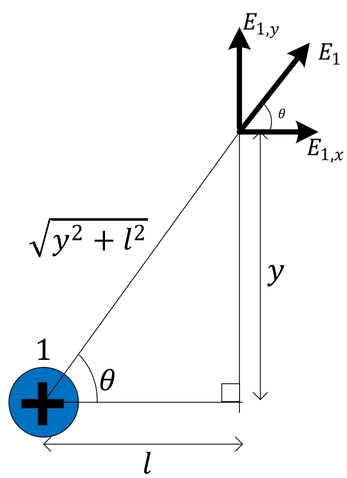
The net electric field at a point y on the vertical axis can be computed using vector addition as we did in example 1. However, in this case we start by noting the symmetry of the scenario to reduce the required computation. Since the charges are equal and the distance to any point on the y axis is the same for each charge, we can write the following relationships.

$$E_{x2} = -E_{x1} \qquad\qquad E_{y2} = E_{y1}.$$

The net electric field can then be expressed as follows.

$$\boldsymbol{E_{net}} = \langle E_{x1}, E_{y1}\rangle + \langle E_{x2}, E_{y2}\rangle$$
$$\boldsymbol{E_{net}} = \langle E_{x1}, E_{y1}\rangle + \langle -E_{x1}, E_{y1}\rangle$$
$$\boldsymbol{E_{net}} = \langle 0, 2E_{y1}\rangle$$

The y component is then found below.



$$E_{y1} = \left(\frac{kq}{y^2 + l^2}\right) sin(\theta)$$
$$E_{y1} = \left(\frac{kq}{y^2 + l^2}\right)\frac{y}{\sqrt{y^2 + l^2}}$$
$$E_{y1} = \left(\frac{kqy}{(y^2 + l^2)^{3/2}}\right)$$

Therefore, for any points on the y axis the electric field has a magnitude given as follows.

$$|\boldsymbol{E_{net}}| = \frac{2kq|y|}{(y^2 + l^2)^{3/2}}$$

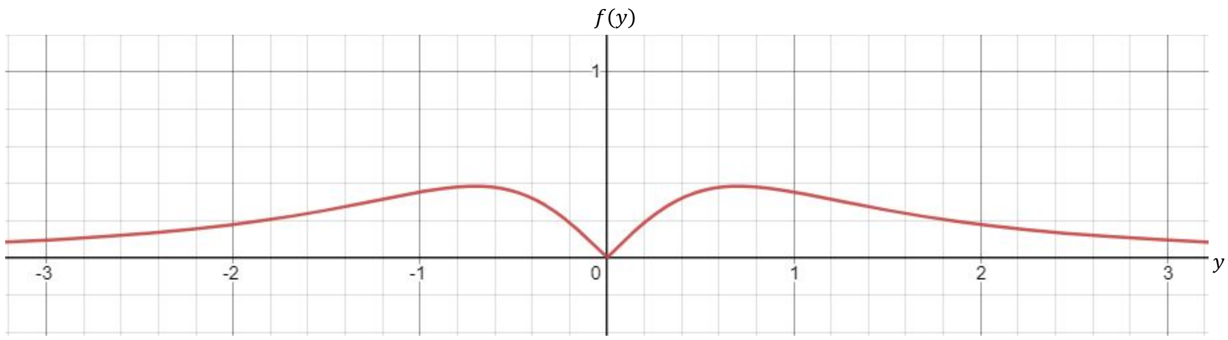
It interesting to evaluate the field when $l = 0$, i.e., a charge of $2q$ located at the origin.

$$|\boldsymbol{E_{net}}| = \frac{2kq|y|}{(y^2 + 0)^{3/2}} = \frac{2kq|y|}{y^3} = k\frac{2q}{y^2}$$

Exactly as we would expect for a single charge of $2q$!

2b: The relative shape of the magnitude of the electric field is shown below using the following general equation.

$$f(y) = \frac{|y|}{(y^2 + 1^2)^{3/2}}$$



Therefore, there should be two maximum values (above and below the x-axis). To find these locations we can differentiate the function and set it to zero.

$$\frac{d}{dy}\big(E(y)\big) = 0$$

$$\frac{d}{dy}\left(\frac{2kqy}{(y^2 + l^2)^{3/2}}\right) = 0$$

$$\frac{2kq(y^2 + l^2)^{3/2} - 2kqy\big(3y(y^2 + l^2)^{1/2}\big)}{(y^2 + l^2)^3} = 0$$

$$\frac{1}{(y^2 + l^2)^{3/2}} = \frac{3y^2}{(y^2 + l^2)^{5/2}}$$

$$3y^2 = \frac{(y^2 + l^2)^{5/2}}{(y^2 + l^2)^{3/2}}$$

$$3y^2 = (y^2 + l^2)$$

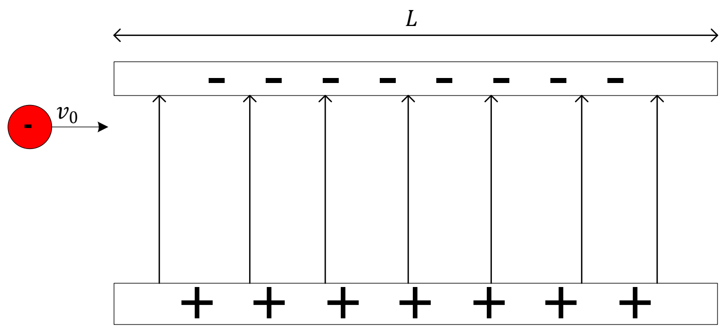
$$2y^2 = l^2$$

$$y = \pm\frac{\sqrt{2}}{2}l$$

Note, the locations are independent of charge size. The value of the electric field at these locations will however be proportional to the charge size.

Example 3: An electron, $m_e = 9.1E^{-31}\ kg$, traveling at $2E^6\ m/s$ enters a $L = 0.1\ m$ region with a uniform electric field of $300\ \ N/C$, as shown below.

a. Find the acceleration of the electron while in the electric field. Find the vertical displacement of the electron while in the electric field.
b. A muon is an elementary particle with the same charge as an electron but with a larger mass, $m_m = 1.9E^{-28}\ kg$. Assuming a muon travels into the electric field with the same initial speed as the electron, find its vertical displacement while in the electric field.



Solution 3a: When the electron, (or muon), enters the region of the electric field it is subjected to an electric force given by

$$\boldsymbol{F_E} = q\boldsymbol{E}$$

The direction of the electric field, i.e., the positive y direction, is the direction that a *positive* charge would feel the force. In this case, since the electron, (or muon), is negative the force on it is directed in the negative y direction. Since there is no force acting in the horizontal direction, the initial speed in this direction remains constant. We can find the acceleration of the particle in the y direction using Newton's 2nd law.

$$a_y = \frac{\sum F_y}{m}$$

$$a_y = \frac{qE}{m}$$

In the case of the electron, we have

$$a_{y,e} = \frac{1.6E^{-19} \cdot 300}{9.1E^{-31}} = \ 5.27E^{13}\ m/s^2$$

To find the vertical displacement we can use kinematics. First, we find the time it takes for the particle to horizontally travel the distance $L = 0.1\ m$. Since the speed is constant in the horizontal direction, we have

$$t = \frac{L}{v_0} = \frac{0.1}{2E^6} = 5E^{-8}\ s$$

Next, we find the vertical distance traveled in this time period.

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}(5.27E^{13})(5E^{-8})^2 = 0.066\ m$$

3b. We can use the same procedure to find the vertical distance a muon would travel. Using the general expression for the acceleration we have

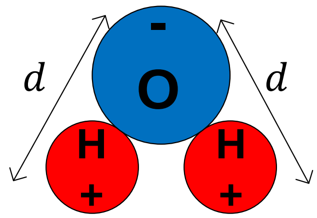
$$a_{y,m} = \frac{qE}{m_m}$$

Since the muon travels the same constant speed in the horizontal direction it takes the same amount of time to get through the region as the electron, i.e., $t = L/v_0$. Finally, we can use the acceleration equation from above to find the vertical displacement of the muon.

$$\begin{aligned}\Delta y &= \frac{1}{2}at^2 \\ &= \frac{1}{2}\left(\frac{qE}{m_m}\right)\left(\frac{L}{v_0}\right)^2 \\ &= \frac{1}{2}\left(\frac{1.6E^{-19}\cdot 300}{1.9E^{-28}}\right)\left(\frac{0.1}{2E^6}\right)^2 \\ &= 0.000316\ m\end{aligned}$$

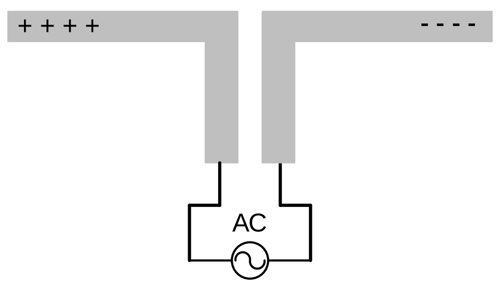
As you can see, since the muon has a much greater mass its vertical displacement is much less.

Example 4: Electric Dipoles - Any system that consists of two equal charges of opposite sign separated by a distance, d, is referred to as an *electric dipole*. Many examples of electric dipoles can be found in nature. A water molecule, H_2O, is an electric dipole. The hydrogen side of the molecule is always somewhat more positive than the oxygen side.

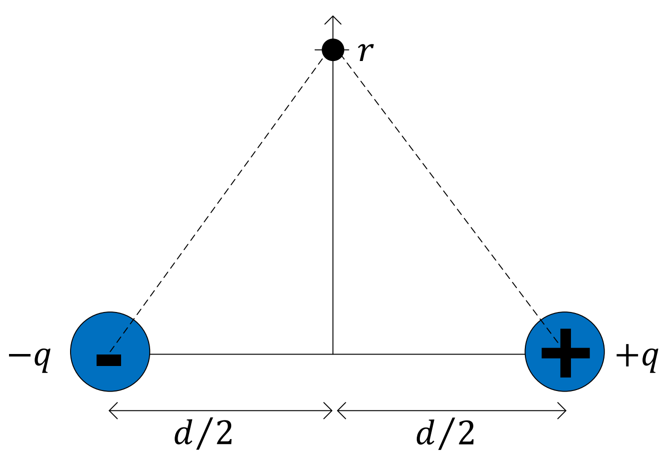


This polar nature of water molecules allows them to bond to each other and is largely responsible for the high surface tension of water.

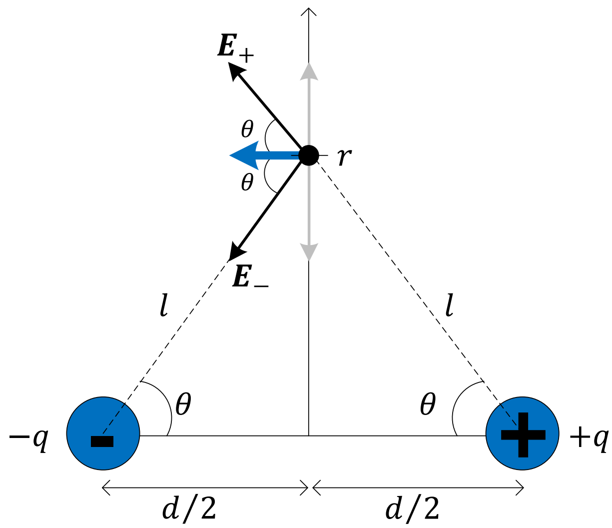
One instance where electric dipoles are used in practice is in the area of telecommunications, where the most widely used class of antenna is a *dipole antenna*. The basic structure consists of two identical conducting elements. From a transmitting antenna perspective, an alternating electric current drives the two elements such that each will be oppositely charged at all times. For reasons outside the scope of the current lesson this alternating of charges allows energy to be radiated from the antenna and ultimately received by another antenna some distance away.



Given the importance of electric dipoles let's determine the electric field that is produced by an electric dipole. The figure below shows two charges, $+q$, and $-q$ that are separated by a distance d. We will find the electric field at any point, r, on the perpendicular bisector of the dipole as shown.



Solution 4: The figure is redrawn showing the E field vectors a distance r above the dipole.



Decomposing these vectors, we see that the vertical components, (shown in light gray), cancel, and the horizontal components, (shown in blue), add. The magnitude of the electric field is then the sum of the horizontal components.

$$E = |E_{+,x}| + |E_{-,x}|$$

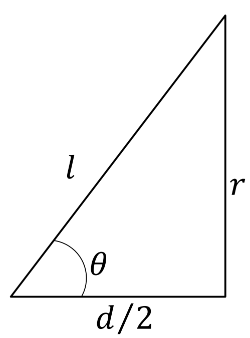
And since $|E_{-,x}| = |E_{+,x}|$

$$E = 2|E_{+,x}|$$

$$E = 2E_+ \cos(\theta)$$

$$E = 2\frac{kq}{l^2}\cos(\theta)$$

Using the right triangle formed by the perpendicular bisector we can express l^2 and $cos(\theta)$ as shown below.



$$l^2 = r^2 + {d^2}/{4}$$

$$\cos(\theta) = \frac{{d}/{2}}{\sqrt{r^2 + {d^2}/{4}}}$$

Substituting we have

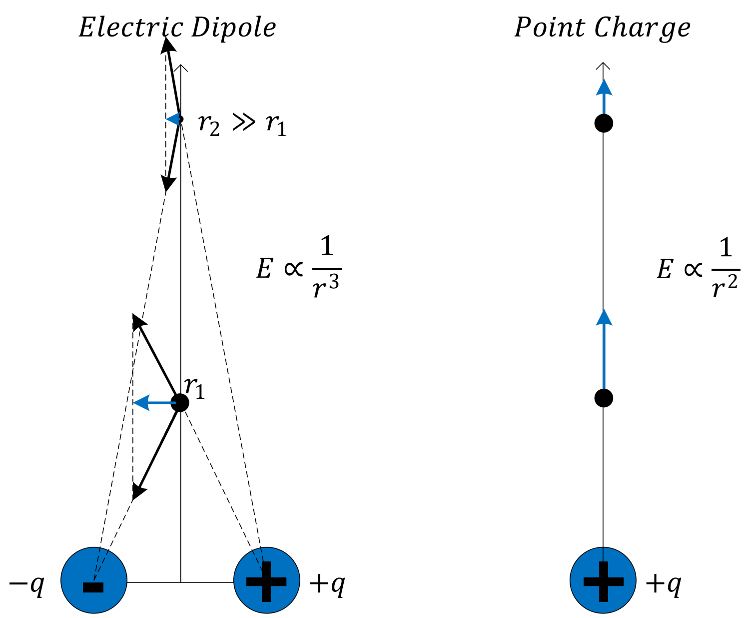
$$E = 2\frac{kq}{\left(r^2 + {d^2}/{4}\right)}\left(\frac{{d}/{2}}{\sqrt{r^2 + {d^2}/{4}}}\right)$$

$$E = k\frac{qd}{\left(r^2 + {d^2}/{4}\right)^{3/2}}$$

Lastly, we consider the case when $r >> d$, for which we have the following.

Electric Field at a distance r, due to an Electric Dipole with separation distance d.	$E = k\frac{qd}{r^3}$	$r >> d$

The important thing to notice here is that compared to a point charge, which decreases as $1/r^2$, the electric field of an electric dipole decreases as $1/r^3$. The physical reason for this is because at large distance the two charges seem to (almost) coincide. Being equal but opposite charges, they appear to (almost) cancel each other.



Lastly, although we derived this result for points along the perpendicular bisector, the magnitude of the electric field varies as $1/r^3$ for all points that are a distance r from the center of the dipole.

Electric Field for Continuous Charge Distributions:

As shown in the examples above, the electric field at a point P from various point charges is computed using the following vector sum

$$\boldsymbol{E}_p = \sum_{i=1}^{N} k \frac{q_i}{\left(r_{ip}\right)^2} \hat{\boldsymbol{r}}_{ip}$$

However, in many cases we would like to find the electric field due to an extended charged object. If we assume the charge is distributed continuously throughout the object, we can divide the charge into infinitesimal charges, dq. The contribution to the electric field from this charge element is then

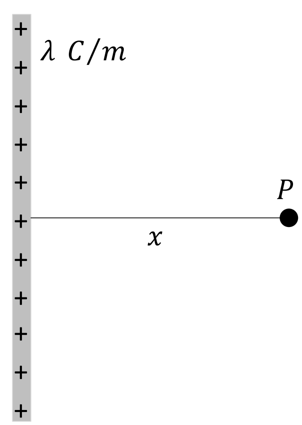
$$d\boldsymbol{E} = k \frac{dq}{r^2} \hat{\boldsymbol{r}}$$

Therefore, the electric field at any point is obtained by integrating over all the infinitesimal charge elements, dq.

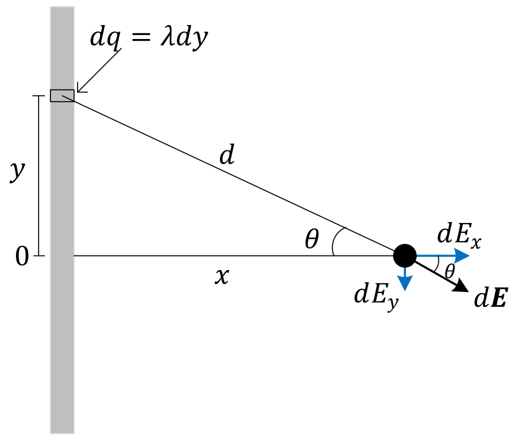
$$\boldsymbol{E} = \int d\boldsymbol{E}$$

In general, this integral is not easy to compute. However, as we will show with the next few examples, some cases are indeed feasible and can provide intuition that is worth the effort. Note, if you are not familiar with calculus you can skip this section

Example 5: Infinite Line of Charge - Determine the magnitude of the electric field at a point P that is a distance x from a very long wire with a uniformly distributed charge. Let λ be the charge per unit length of the wire in C/m.



Solution 5: To compute the electric field from a continuous charge distribution we start by identifying an infinitesimal charge element, dq, and its contribution to the electric field, $d\boldsymbol{E}$, at a given point. The figure below, which places the center of the wire at $y = 0$, illustrates this.



First thing to notice is that as we sum the electric field elements, $d\boldsymbol{E}$, the y components above the x-axis will completely cancel the y components from below the x-axis. Therefore, we can consider the x components only, which can be written as follows.

$$dE_x = |d\boldsymbol{E}|\,cos(\theta)$$

$$dE_x = k\frac{dq}{d^2}cos(\theta)$$

$$dE_x = k\frac{\lambda dy}{d^2}cos(\theta)$$

$$dE_x = k\lambda\left(\frac{1}{d^2}\right)cos(\theta)\,dy$$

If we consider the case where x is much smaller than the length of the wire, the wire can be considered of infinite length. Therefore, we should integrate for $-\infty < y < \infty$. However, the integration will be simpler if we instead equivalently integrate over θ, i.e., $-\pi/2 < \theta < \pi/2$. To do so we must first change the integration variable from y to θ, which we can do using the following substitutions.

$$y = x\,tan(\theta) \rightarrow dy = \frac{x}{cos^2(\theta)}d\theta \qquad\qquad d = \frac{x}{cos(\theta)} \rightarrow \frac{1}{d^2} = \frac{cos^2(\theta)}{x^2}$$

Now we can rewrite the electric field contribution using these substitutions.

$$dE_x = k\lambda\left(\frac{cos^2(\theta)}{x^2}\right)cos(\theta)\frac{x}{cos^2(\theta)}d\theta$$

$$dE_x = \frac{k\lambda}{x}cos(\theta)\,d\theta$$

Finally, we integrate to find the total electric field. Note that since the wire is of infinite length the electric field is a function of x only.

$$E = \frac{k\lambda}{x}\int_{-\pi/2}^{\pi/2} cos(\theta)\, d\theta$$

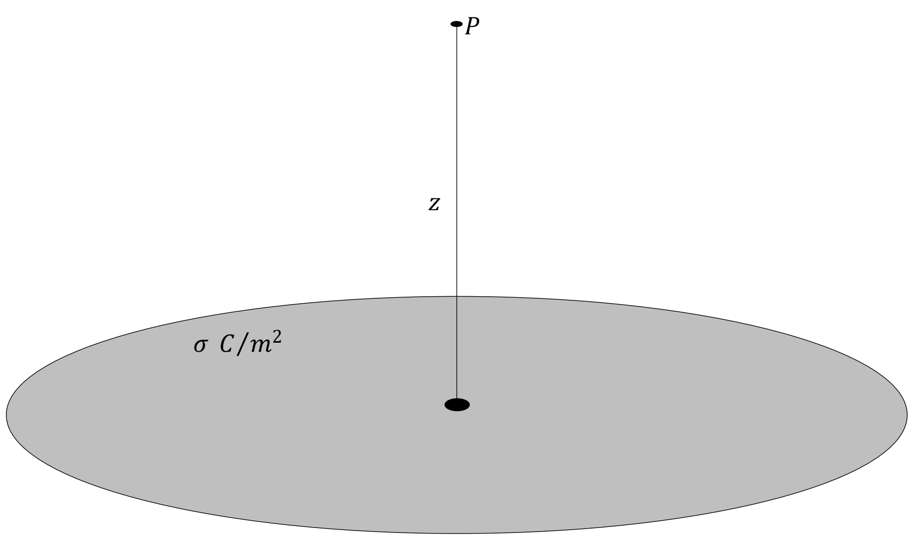
$$E = \frac{k\lambda}{x}\left(sin(\pi/2) - sin(-\pi/2)\right)$$

$$E = 2k\frac{\lambda}{x}$$

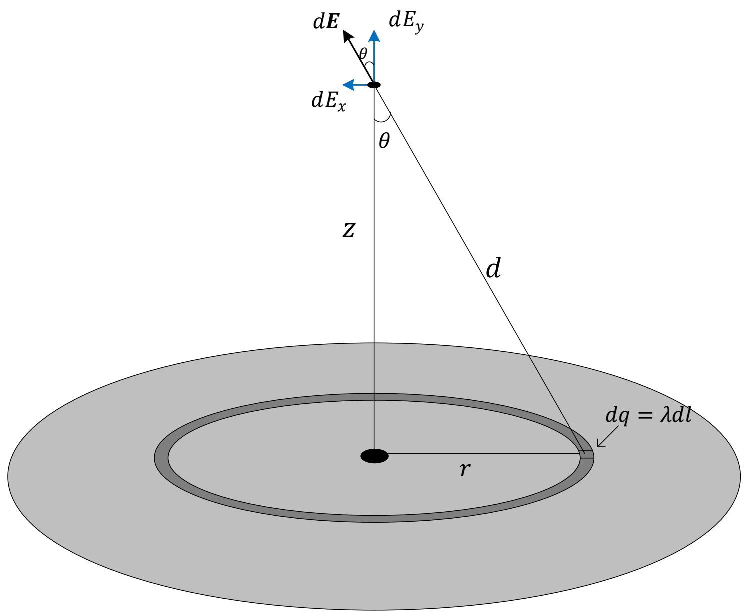
Electric Field at a distance x, due to an infinite length wire with charge density, λ.	$E = 2k\frac{\lambda}{x}$

Recall that the magnitude of the electric field from a point charge dropped off as $1/x^2$, while the magnitude dropped off as $1/x^3$ for an electric dipole. In this case, we see that that for an infinite wire, the magnitude drops off as $1/x$. Although the length of the wire was considered infinite, the results holds as long as x is small compared to the length of the wire.

Example 6: Infinite Plane of Charge - Determine the magnitude of the electric field at a point P above an infinite plane with a uniformly distributed charge. Let σ be the charge per unit area in C/m^2.



Solution 6: To solve this problem we will begin by finding the electric field from a single ring of charge as shown below. We assume the ring has a radius, r, and a charge density, $\lambda\ C/m$.



In this case, we notice that the x component of the electric field will cancel as we move around the ring. Therefore, we can consider the y components only, which can be written as follows.

$$dE_y = |d\mathbf{E}|\, cos(\theta)$$
$$dE_y = k\frac{\lambda dl}{d^2} cos(\theta)$$
$$dE_y = k\lambda\frac{1}{d^2} cos(\theta)\, dl$$

Since we would like to find the electric field as a function of z, we will first express $cos(\theta)$ and d as functions of z as shown below.

$$d^2 = z^2 + r^2 \qquad\qquad cos(\theta) = \frac{z}{d} = \frac{z}{\sqrt{z^2 + r^2}}$$

Substituting and integrating over l from 0 to $2\pi r$, we have the following.

$$dE_y = k\frac{\lambda}{z^2 + r^2}\left(\frac{z}{\sqrt{z^2 + r^2}}\right) dl$$
$$dE_y = k\frac{\lambda z}{(z^2 + r^2)^{3/2}} dl$$
$$E = \int_{l=0}^{l=2\pi r} dE_y$$
$$E = \int_0^{2\pi r} k\frac{\lambda z}{(z^2 + r^2)^{3/2}} dl$$
$$E = k\frac{\lambda z 2\pi r}{(z^2 + r^2)^{3/2}}$$

Lastly, we note that the total charge along the ring is given by $q = \lambda 2\pi r$, which gives us the following equation for the total electric field z meters above a ring of charge with radius r and a total charge of q.

Electric Field at a distance z, due to a ring of wire with radius, r, and total charge, q.	$E = k\dfrac{qz}{(z^2 + r^2)^{3/2}}$

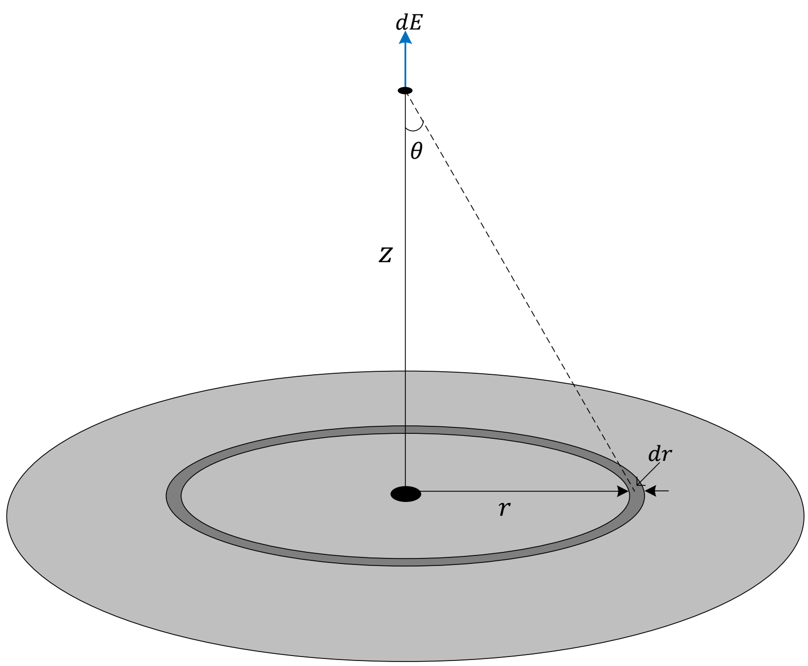
Note, when $z >> r$, we have

$$E = k\frac{qz}{(z^2)^{3/2}} = k\frac{q}{z^2}$$

Which is the equation for the electric field of a point charge, since at far distance the ring looks like a single point.

This brings us back to the original question, i.e., "*Determine the magnitude of the electric field at a point P above an infinite plane with a uniformly distributed charge. Let σ be the charge per unit area in C/m^2.*"

We can now begin to answer this question using the previous result. We start by redrawing the figure form above.



In this case, we consider the ring of charge as our infinitesimal charge element, dq. The contribution to the electric field this ring of charge makes is exactly what we computed above.

$$dE = k\frac{z\,dq}{(z^2 + r^2)^{3/2}}$$

The infinitesimal area of the ring is given by the circumference, $2\pi r$, multiplied by the thickness, dr, i.e., $dA = 2\pi r dr$. Using the charge density, σ, the charge contained in the ring is $dq = \sigma 2\pi r dr$. Therefore, the electric field due to the ring can be rewritten as follows.

$$dE = k\frac{z\sigma 2\pi r}{(z^2+r^2)^{3/2}}dr$$

The total electric field is then found by integrating for $0 < r < \infty$.

$$E = \lim_{R\to\infty}\left(kz\sigma 2\pi \int_0^R \frac{r}{(z^2+r^2)^{3/2}}dr\right)$$

To solve this integral we can use the following u-substitution.

$$u = z^2 + r^2 \quad \to \quad du = 2rdr$$

$$\begin{aligned} kz\sigma 2\pi \int_0^R \frac{r}{(z^2+r^2)^{3/2}}dr &= kz\sigma\pi \int_{z^2}^{z^2+R^2} \frac{1}{(u)^{3/2}}du \\ &= kz\sigma\pi \left(\frac{-2}{(u)^{\frac{1}{2}}}\right)_{z^2}^{z^2+R^2} \\ &= 2kz\sigma\pi\left(\frac{1}{z} - \frac{1}{(z^2+R^2)^{\frac{1}{2}}}\right) \\ &= 2k\sigma\pi\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \end{aligned}$$

The electric field for an infinite plane is then given as shown.

$$E = \lim_{R\to\infty}\left(2k\sigma\pi\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\right)$$
$$E = 2k\sigma\pi$$

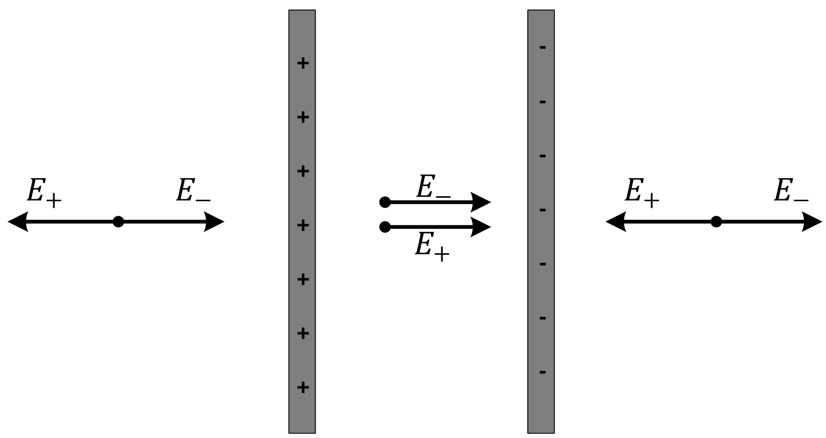
Furthermore, k can be written as $\frac{1}{4\pi\epsilon_o}$, where $\epsilon_o = 8.854E^{-12}$ is the permittivity of free space. Finally, we can write the electric field for an infinite plane as follows.

Electric Field due to an infinite plane of charge with a charge density, σ.	$E = \frac{\sigma}{2\epsilon_o}$

As you can see, electric field from an infinite plane is constant for any point above (or below) the plane. Even without such an infinite plane, we can still use this result as a very good approximation for computing the E field close to a plane of finite size, as we discuss below.

An important component which is a part of most electrical circuits is a capacitor. A capacitor is essentially two thin parallel plates with opposite charge that are separated by a distance that is very small compared to the size of the plates. If we approximate the plates as being infinite in size, we can use the results from above to find the electric field both inside and outside of the capacitor. Since the magnitude of the electric field from each plate is constant, we can see from the figure that the field cancels outside of the capacitor and adds inside.

Electric field inside and outside of a parallel plate capacitor	
$E_{outside} = \frac{\sigma}{2\epsilon_o} - \frac{\sigma}{2\epsilon_o} = 0$	$E_{inside} = \frac{\sigma}{2\epsilon_o} + \frac{\sigma}{2\epsilon_o} = \frac{\sigma}{\epsilon_o}$



Final Summary for Electric Field

Electric Field Lines

The electric field is a vector field that permeates the space around charged objects, and that has units of force per charge (N/C). The direction of each vector is chosen based on the direction of the force that would act on a *positive* test charge placed in this field.

Instead of drawing the individual vectors we draw so called *field lines*. We can list three rules for drawing electric field lines:

1. Field lines begin and end at electric charges (or at infinity).
2. At any point in space the net electric force vector is tangent to the electric field line at that point and it points in the same direction.
3. Field lines can never intersect each other. (Otherwise at the point of intersection we could draw two different tangent lines indicating two different directions for the net force, which is not physically possible.)

Electric Field for Point Charges

The electric field at a point p, that is a distance r from a single point charge, q, is given as:

$$\boldsymbol{E}_p = \frac{kq}{r^2}\hat{r}_{qp} \quad N/C$$

The electric field at a point p from N point charges is given as a vector sum.

$$\boldsymbol{E}_p = \sum_{i=1}^{N} k\frac{q_i}{\left(r_{ip}\right)^2}\hat{\boldsymbol{r}}_{ip}$$

The electric force on a charge, q, placed at a point p in an electric field is given as:

$$\boldsymbol{F}_E = \boldsymbol{E}_p q$$

Electric Dipole

Any system that consists of two equal charges of opposite sign separated by a distance, d, is referred to as an *electric dipole.*

The electric field at a distance r from an Electric Dipole with separation distance d, is given as

$$E = k\frac{qd}{r^3}, \quad for\, r >> d$$

Electric Field for Continuous Charge Distributions:
Infinite Length Wire
The electric field a distance x from an infinite length wire with charge density, λ, is given as: $$E = 2k\frac{\lambda}{x}$$
Infinite Plane
The electric field due to an infinite plane of charge with a charge density, σ, is given as: $$E = \frac{\sigma}{2\epsilon_o}$$
Parallel Plate Capacitor
$$E_{outside} = 0, \qquad E_{inside} = \frac{\sigma}{\epsilon_o}$$

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