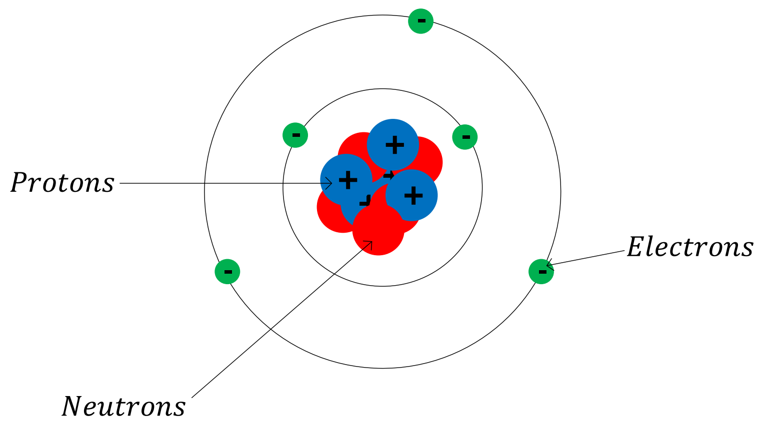
Physics 2 E&M – Electric Charge

The notion of static electricity is familiar to most of us from everyday experience. For example, can you recall a time when you felt a "shock" from touching a metal object after walking across a carpeted floor? How about rubbing a balloon against your head and making it "stick" to the wall? These seemingly common experiences are the result of the phenomenon of static electricity. In this lesson we will take a closer look at this phenomenon to understand exactly what is happening.

As we know, all matter if made of atoms. Atoms, as it turns out, are made from three fundamental particles: protons, neutrons, and electrons. Two of these particles, the proton, and the electron, have what we call an *electric charge*. Neutrons, on the other hand, have no charge, i.e., they are neutral particles. The electric charge is what is ultimately responsible for the phenomenon of static electricity. The figure below illustrates an atom, which consists of protons, which are *positively charged*; electrons, which are *negatively charged*; and neutrons with no charge. In their normal state atoms have the same number of electrons and protons, therefore have no *net charge*. If we could somehow add or remove some number of protons and/or electrons such that there is no longer the same number, the atom would then have a *net charge*. Protons (and neutrons) are held very tightly in the center of the atom, called the nucleus. Removing or adding a particle in the nucleus of an atom is considered a *nuclear* reaction, which requires a considerable amount of energy to accomplish. Electrons, on the other hand, are much easier to add or remove from an atom. The exchange of electrons is what takes place in chemical reactions. When electrons are added or removed from an atom it becomes electrically charged, (negatively charged if electron(s) are added and positively charged if electron(s) are removed). Objects that are electrically charged interact with each other, which means they exert forces each other, in this case without the need to be in direct contact.

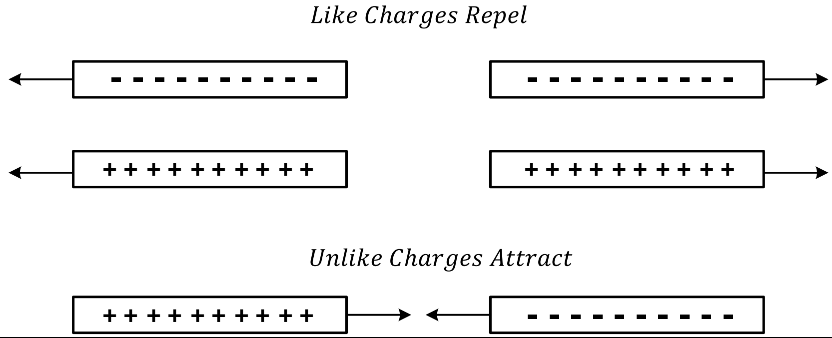


Understanding the fundamental source of electric charge, i.e., a surplus/deficit of electrons, will allow us to solve problems related to static electric charge in macroscopic objects, since all objects are made of atoms.

Most of us are likely familiar with the law of electrical charges stated below.

Law of Electric Charged Objects

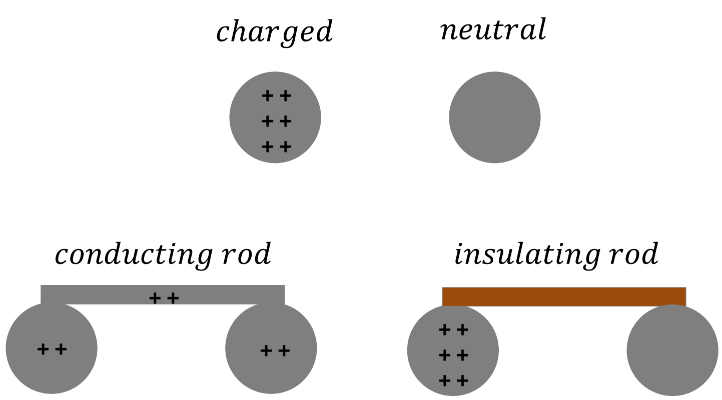
- Like charged objects repel each other
- Unlike charged objects attract each other



Insulators and Conductors

As we know, any macroscopic material object is made of an immense number of atoms. Certain materials are made from atoms that have some electrons that are free to move from atom to atom within the object. These materials, (e.g., metals), are referred to as *conductors.* For other materials, referred to as *insulators*, the electrons are bound much more tightly to the nucleus of the atoms are not free to move (e.g., rubber, wood). Imagine we have two metal spheres; one is positively charged, i.e., electrons were removed, and the other is neutral. Now let's perform two different experiments and observe the results.

1. We simultaneously touch the two spheres with a *metal* rod.
   - The spheres, together with the metal rod are now considered a single conducting material. Therefore, electrons are free to move throughout and will do so until a uniform charge distribution is reached. The entire object is then positively charged.
2. We simultaneously touch the two spheres with a *wooden* rod.
   - Since the wooden rod is an insulator with no "free" electrons, the charge on the two spheres remains unchanged.



Charge by Friction, Conduction, and Induction

In the example from the last section, we started with a charged sphere, but we didn't discuss how this sphere became charged in the first place. In this section we examine how to charge an object. We can identify three ways to do this; by friction, by conduction, or by induction.

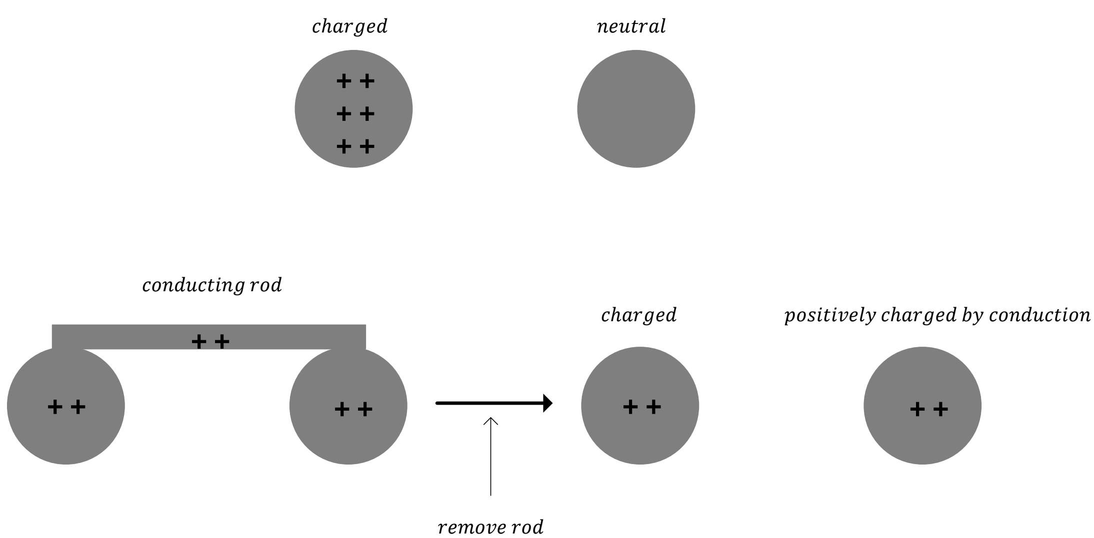
Charge by Friction

Most of us have first-hand experience of this type of charging. For example, we may have walked across a carpet, then touched a metal object and felt a "shock". As you walked across the carpet your shoes rubbed against it, causing some electrons from the carpet to collect on your body, thereby making you negatively charged (and leaving the carpet positively charged). When you subsequently touch a conducting surface the extra electrons from your body move onto the conductor causing the shock sensation you feel. Charging by friction is useful for charging insulating materials (Note the rug, an insulator, is left with a positive charge).

Charge by Conduction

The experiment we performed in the previous section with the two metal spheres is an example of charging by conduction. Recall we started with one positively charged sphere and one neutral sphere. When we connected these spheres via a conducting object, electrons were redistributed leaving the two spheres and the rod all with a positive charge. If we then remove the rod, the previously neutral sphere remains positively charged. The sphere has been charged via conduction. Charging by conduction is also referred to as charging by "contact".

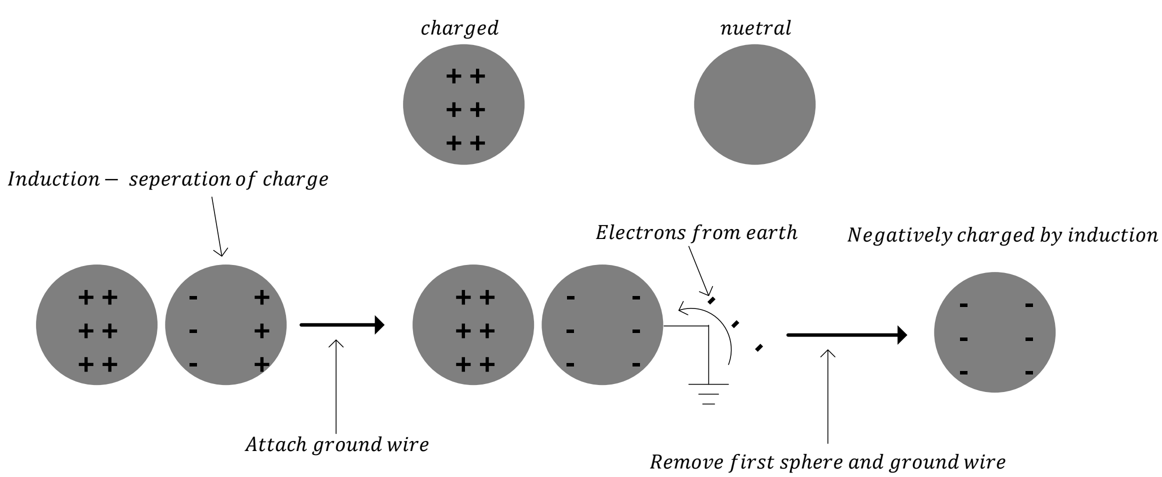
Charge by Conduction (Contact)



Charge by Induction

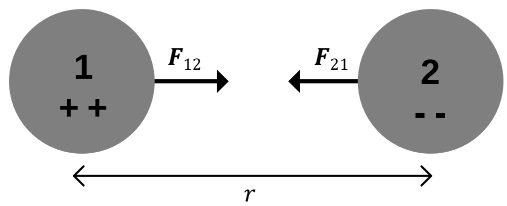
Suppose we again start with the same two spheres. This time we bring the spheres close to each other, but **do not** allow *contact.* Electrons in the neutral sphere, which are free to move within the sphere, will move towards the positively charged sphere, leaving the other side of the sphere with a positive charge. This will cause the sphere to become what we refer to as a polar object: "*A neutral object that has its charge distributed non-uniformly*". Although the entire sphere is still neutral, we can say that a charge has been *induced* at opposite ends. If we were to break the sphere in half, we would have two charged objects, one positive and one negative. Another, more convenient way to end up with a single charged object is to connect one side of the polar sphere with a conducting wire to the earth (called ground). The earth is so large that it can easily accept or give up electrons. In this case electrons would move from the earth to the sphere. After removing both the positively charged sphere and the ground wire the second sphere will remain negatively charged. Hence, we have created a charge in the previously neutral sphere via *induction*.

Charge by Induction



Coulomb's Law

As mentioned, electrically charged objects exert forces on one another (conveniently called electrical forces). The question then becomes: Can we quantify this force? Charles Coulomb investigated this question and found that the behavior of the electrical force is similar to the gravitational force; the main difference being that the electrical force can be both attractive and repulsive. Let's take the two oppositely charged objects as shown below.

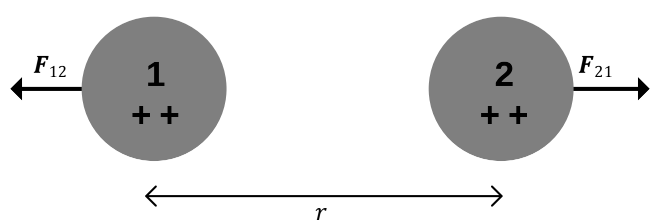


The force on object 1 by object 2, $\boldsymbol{F}_{12}$, is equal in magnitude but opposite in direction to the force on object 2 by object 1, $\boldsymbol{F}_{21}$. The magnitude of this force is given by:

$$F_E = k\frac{Q_1 Q_2}{r^2}$$

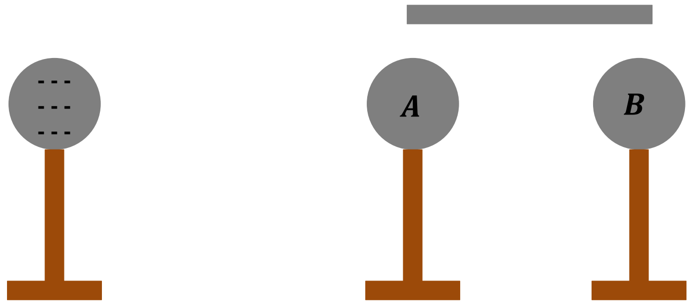
Where, Q_1 is the magnitude of the charge on object 1, Q_2 is the magnitude of the charge on object 2, r is the distance between the objects, and k is a proportionality constant given by: $9E^9\ N \cdot m^2/C^2$. The unit of charge is the coulomb, C.

On the other hand, if, as in the figure below, the objects were both positively charged, (both negatively charged would be the same), we would have a repulsive force and the force vectors would be directed as shown below. Of course, the magnitude of the force would be the same as above.



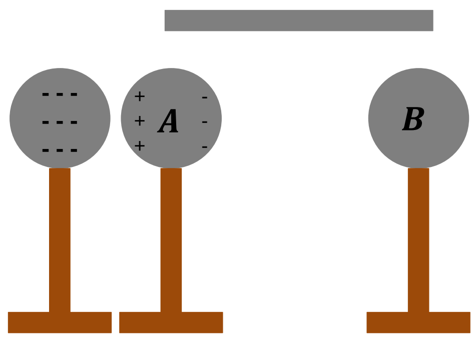
Let's do some examples to solidify our understanding of charged objects and how they interact with each other, i.e., exert forces on each other.

Example 1: Two uncharged metal balls, $\boldsymbol{A}$ and $\boldsymbol{B}$, stand on insulating rods. A third ball, carrying a negative charge, is brought near ball $\boldsymbol{A}$. A conducting wire is then placed between $\boldsymbol{A}$ and $\boldsymbol{B}$ and then removed. Finally, the third ball is removed. What are the charges on balls $\boldsymbol{A}$ and $\boldsymbol{B}$ at the end of the experiment?

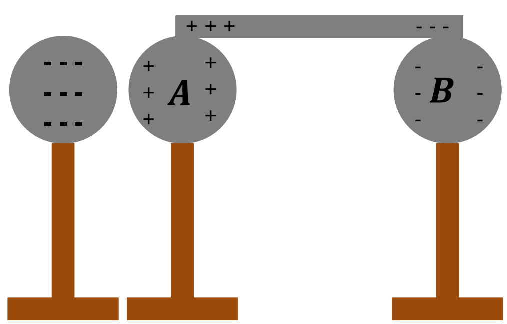


Solution 1: The experiment and its consequences are listed in order below.

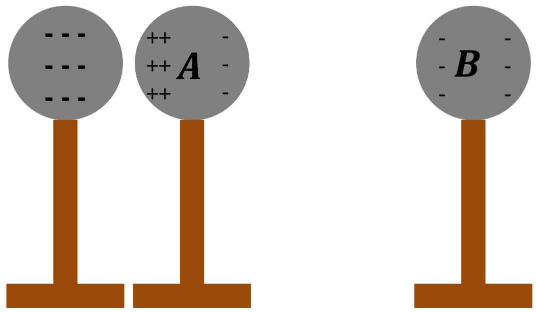
- The negatively charged ball is brought near ball $\boldsymbol{A}$.
  - Electrons in ball $\boldsymbol{A}$ are forced to the opposite side, creating a polar object.



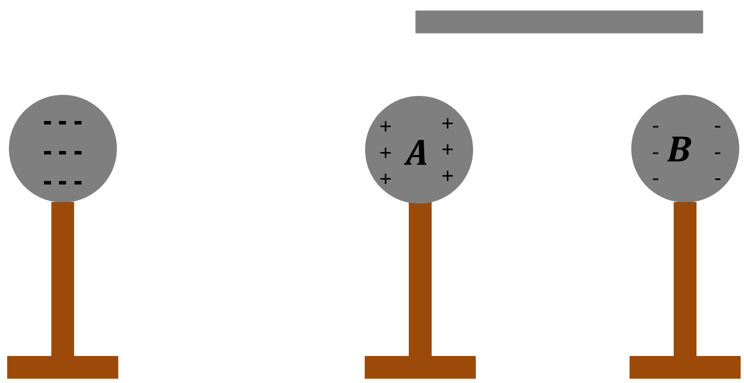
- A conducting wire is placed between $\boldsymbol{A}$ and $\boldsymbol{B}$.
  - Ball $\boldsymbol{A}$ and $\boldsymbol{B}$, along with the conducting wire, can be treated as a single object. In this case, electron will continue to be pushed away from the left side of this larger object. The entire object is now polar with negative charge on the right, i.e., ball $\boldsymbol{B}$, and positive charge on the left, i.e., ball $\boldsymbol{A}$.



- The conducting wire is removed.
  - The excess charge, positive on ball $\boldsymbol{A}$ and negative on ball $\boldsymbol{B}$, will remain. Since the negatively charge ball is still near ball $\boldsymbol{A}$, it will again become polarized, but will still remain with a net positive charge.



- The third negatively charged ball is removed.
  - Ball $\boldsymbol{A}$ will be unpolarized but will remain positively charged.
  - Ball $\boldsymbol{B}$ will remain negatively charged.
  - Ball $\boldsymbol{A}$ was charge by induction and ball $\boldsymbol{B}$ was charge by conduction.



Example 2: Assume the third negatively charged sphere from the previous example was measured to have a charge of $-1.0E^{-5}\ C$. How many excess electrons make up this charge? The magnitude of the charge for an electron, (and a proton), is $1.6E^{-19}\ C$.

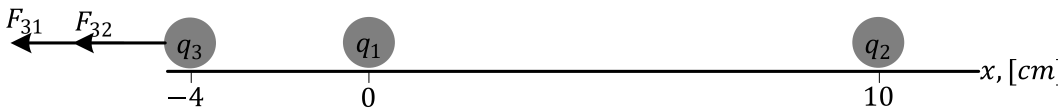
Solution 2: Each excess electron contributes, $q_e = -1.6E^{-19}\ C$, of change. Therefore, the number of electrons, n_e, required to create a total charge, $q_T = -1.0E^{-5}\ C$, is given as follows.

$$n_e = \frac{q_T}{q_e} = \frac{-1.0E^{-5}}{-1.6E^{-19}} = 6.25E^{13}\ excess\ electrons$$

Example 3: Two particles with charge, $q_1 = 5.0\ uC$ and $q_2 = 10.0\ uC$ are placed on the x-axis as shown below. Find the force on a third particle with a charge of $q_3 = 1.0\ uC$ when it is placed in the following three locations on the x-axis: $x_1 = -4cm,\ \ x_2 = 14\ cm, x_3 = \ 5\ cm.$ Then find the point on the x-axis where the net force on q_3 is zero.

Solution 3:

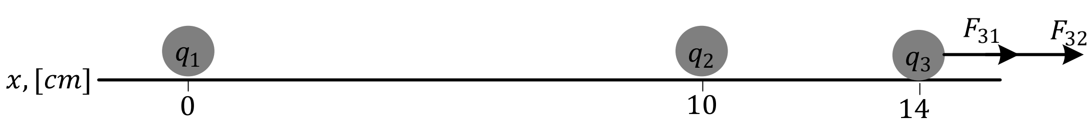
Position 1: Since all three charges are positive, both forces on q_3 are repulsive and point to the left as shown below.



The magnitude of the net force on q_3 is given by the sum the forces from q_1 and q_2.

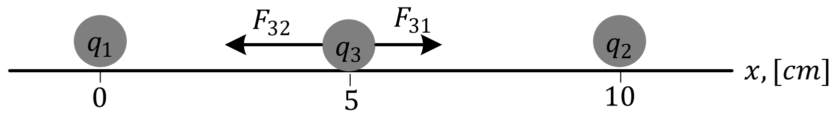
$$\begin{aligned}
F_{net} &= F_{31} + F_{32} \\
&= \frac{kq_3q_1}{(d_{31})^2} + \frac{kq_3q_2}{(d_{32})^2} \\
&= kq_3\left(\frac{q_1}{(d_{31})^2} + \frac{q_2}{(d_{32})^2}\right) \\
&= 9E^9 \cdot 1E^{-6}\left(\frac{5E^{-6}}{(-0.04)^2} + \frac{10E^{-6}}{(0.14)^2}\right) \\
&= 33.7\ N
\end{aligned}$$

Position 2: In this case both forces point to the right.



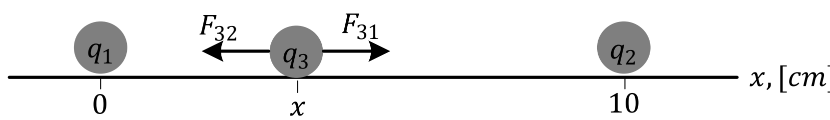
$$\begin{aligned}
F_{net} &= kq_3\left(\frac{q_1}{(d_{31})^2} + \frac{q_2}{(d_{32})^2}\right) \\
&= 9E^9 \cdot 1E^{-6}\left(\frac{5E^{-6}}{(0.14)^2} + \frac{10E^{-6}}{(0.04)^2}\right) \\
&= 58.5\ N
\end{aligned}$$

Position 3: In this case the forces point in opposite directions. Therefore, we subtract the magnitudes to find the net force.



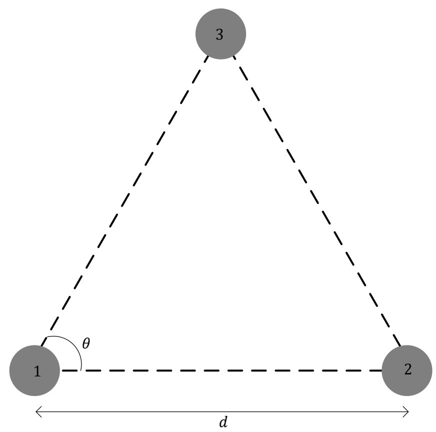
$$\begin{aligned} F_{net} &= F_{31} - F_{32} \\ &= \frac{kq_3 q_1}{(d_{31})^2} - \frac{kq_3 q_2}{(d_{32})^2} \\ &= \frac{q_3}{(d_{31})^2}(q_1 - q_2) \\ &= \frac{9E^9 \cdot 1E^{-6}}{(0.05)^2}(5E^{-6} - 10E^{-6}) \\ &= 18\ N \end{aligned}$$

As you can see from the examples above only when q_3 is placed between the two charges is it possible to have a net force of zero, i.e., the magnitude of the forces are equal. Furthermore, since q_2 is larger than q_1, the location where the forces balance, which we call x, will be closer to q_1 than to q_2. To find this location we set the magnitude of the forces equal and solve for x.

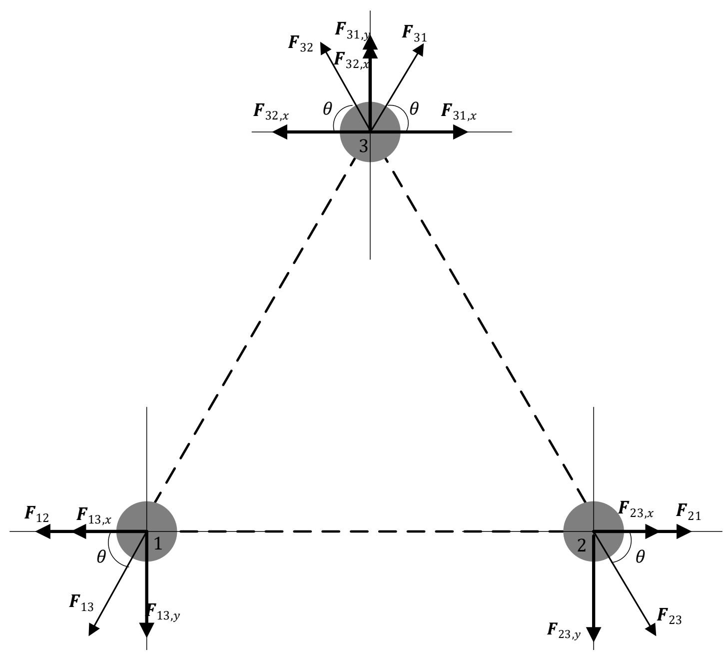


$$\begin{aligned} F_{31} &= F_{32} \\ \frac{kq_3 q_1}{(x)^2} &= \frac{kq_3 q_2}{(10 - x)^2} \\ \frac{q_1}{(x)^2} &= \frac{q_2}{(10 - x)^2} \\ \sqrt{q_1}(10 - x) &= \sqrt{q_2}x \\ x &= \frac{10\sqrt{q_1}}{\left(\sqrt{q_1} + \sqrt{q_2}\right)} \\ x &= \frac{10\sqrt{5E^{-6}}}{\left(\sqrt{5E^{-6}} + \sqrt{10E^{-6}}\right)} \\ x &= 4.14\ cm \end{aligned}$$

Example 4: Three particles, each with a charge of $11.0\ uC$, are located at the corners of an equilateral triangle with a side length of $15\ cm$. Find the magnitude and direction of the net force on each particle.



Solution 4: We start by redrawing the figure, showing the forces on each charge.



The forces on each charge are shown decomposed into their respective x and y components. Similar to Physics 1, we will compute the net force on each charge by first separately computing in the x and y direction. Note, since the charges are arranged at the corners of an equilateral triangle, the distance between any two charges is the same. Furthermore, all angles shown are equal to 60°. Finally, each particle has the same charge, i.e., $q_1 = q_2 = q_3 = q = 11uC$. With this the forces are computed below.

Net Force on Charge 1:

Net force on charge 1 in the x direction

$$F_{1,x} = F_{12,x} + F_{13,x}$$

$$F_{1,x} = k\frac{q^2}{d^2} + k\frac{q^2}{d^2}cos(\theta)$$

$$F_{1,x} = k\frac{q^2}{d^2}(1 + cos(\theta))$$

$$F_{1,x} = 9E^9\left(\frac{11\ E^{-6}}{0.15}\right)^2 (1 + cos(60^{\circ}))$$

$$F_{1,x} = 72.6\ N$$

Net force on charge 1 in the y direction

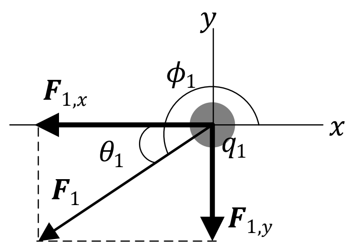
$$F_{1,y} = F_{13,y}$$

$$F_{1,y} = k\frac{q^2}{d^2}sin(\theta)$$

$$F_{1,y} = 9E^9\left(\frac{11\ E^{-6}}{0.15}\right)^2 sin(60^{\circ})$$

$$F_{1,y} = 42\ N$$

We can use the diagram below to find the magnitude and angle from the positive x-axis.



$$F_1 = \sqrt{{F_{1,x}}^2 + {F_{1,y}}^2}$$

$$F_1 = \sqrt{72.6^2 + 42^2}$$

$$F_1 = 84\ N$$

$$\theta_1 = tan^{-1}\left(\frac{F_{1,y}}{F_{1,x}}\right)$$

$$\theta_1 = tan^{-1}\left(\frac{42}{72.6}\right)$$

$$\theta_1 = 30^{\circ} \rightarrow \phi_1 = 180^{\circ} + 30^{\circ} = 210^{\circ}$$

Net Force on Charge 2: Using symmetry, you should be able to see that the magnitude of the forces on charge 2 are the same as for charge 1. We show the component force equations below for completeness.

Net force on charge 2 in the x direction

$$F_{2,x} = F_{21,x} + F_{23,x}$$

$$F_{2,x} = k\frac{q^2}{d^2} + k\frac{q^2}{d^2}cos(\theta)$$

$$F_{2,x} = 72.6\ N$$

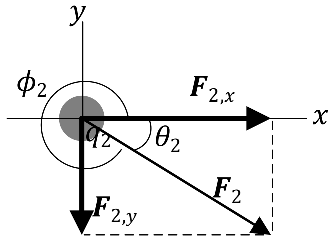
Net force on charge 2 in the y direction

$$F_{2,y} = F_{23,y}$$

$$F_{2,y} = k\frac{q^2}{d^2}sin(\theta)$$

$$F_{2,y} = 42\ N$$

The magnitude and angle can be computed from symmetry.



$$F_2 = 84\ N$$

$$\theta_2 = 30^{\circ} \rightarrow \phi_2 = 360^{\circ} - 30^{\circ} = 330^{\circ}$$

Net Force on Charge 3: We can again use symmetry in terms of the magnitude of the force on charge 3, which should be identical to the previous two charges. For the angle we should notice that the x components cancel leaving a force in the y direction only. However, once again, we show the equations for completeness.

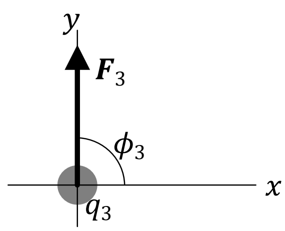
Net force on charge 3 in the x direction

$$F_{3,x} = F_{31,x} - F_{32,x}$$
$$F_{3,x} = k\frac{q^2}{d^2}cos(\theta) - k\frac{q^2}{d^2}cos(\theta)$$
$$F_{3,x} = 0\ N$$

Net force on charge 3 in the y direction

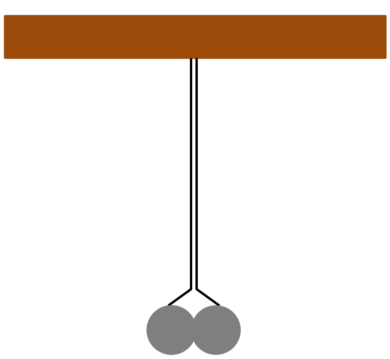
$$F_{3,y} = F_{31,y} + F_{32,y}$$
$$F_{3,y} = k\frac{q^2}{d^2}sin(\theta) + k\frac{q^2}{d^2}sin(\theta)$$
$$F_{3,y} = 2\left(k\frac{q^2}{d^2}sin(\theta)\right)$$
$$F_{3,y} = 2(F_{1,y})$$
$$F_{3,y} = 84\ N$$

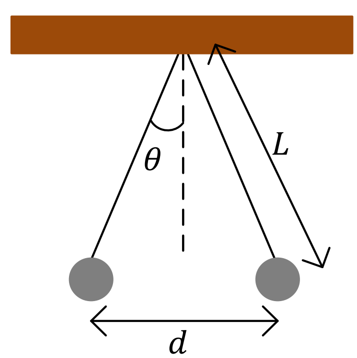
The resulting for diagram is shown below for completeness.



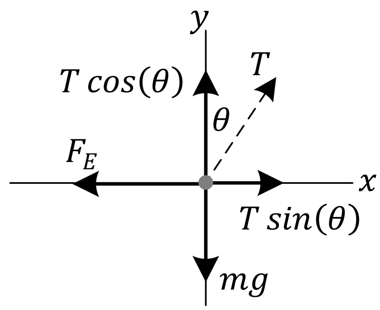
$$F_3 = 84\ N$$
$$\phi_3 = 90^{\circ}$$

Example 5: Two spheres with identical mass, $m = 0.01\ kg$, hang from 20 cm long strings as shown in the first figure below. The spheres are then charged with an identical, but unknown, charge. As a result, the charges separate and come to equilibrium such that the strings make an angle of $\theta = 10^{\circ}$with the vertical. Find the amount of charge on each sphere.





Solution 5: Once charged, the spheres separate because of the mutual electrostatic force. We can draw the free-body diagram for the left sphere as shown below.



Since the sphere is in equilibrium the forces on it must balance. For the x direction we have

$$F_E = T\,sin(\theta)$$
$$\frac{kq^2}{d^2} = T\,sin(\theta)$$

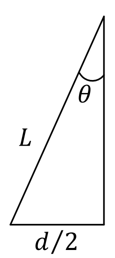
The tension can then be found using the balanced forces in the y direction.

$$T\,cos(\theta) = mg$$
$$T = \frac{mg}{cos(\theta)}$$

Substituting this into the first equation and solving for the charge we have

$$\frac{kq^2}{d^2} = \frac{mg}{cos(\theta)} sin(\theta)$$
$$q = d\sqrt{\frac{mg\,tan(\theta)}{k}}$$

Next, we relate the separation distance to the length of the string using the following right triangle.



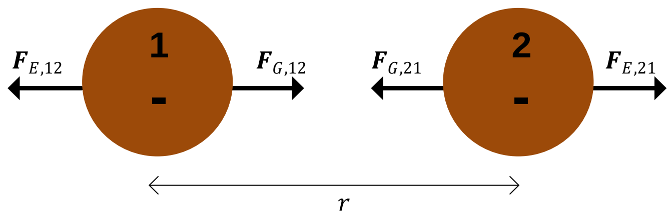
$$sin(\theta) = \frac{d/2}{L} \rightarrow d = 2L\,sin(\theta)$$

Finally, the charge is found as follows.

$$q = 2L\,sin(\theta)\sqrt{\frac{mg\,tan(\theta)}{k}}$$

$$q = 2 \cdot 0.2 \cdot sin(10^\circ)\sqrt{\frac{0.01 \cdot 9.8\,tan(10^\circ)}{9E^9}} = 9.6E^{-8}\,C$$

Example 6: In order to lift an object, one needs to apply a force that is greater than the gravitational force from the earth. Imagine an object being lifted from a table using the electrostatic attractive force. For the object to lift, the electrostatic force must be greater than the opposing gravitational force. The simple example of a charged balloon lifting pieces of paper from a table seems to suggest that the electrostatic force generated from a relatively small balloon is "stronger" than the gravitational force generated from the, much larger, earth. To explore this idea in more detail let's compare the gravitational force to the electrostatic force for two copper atoms placed 1 cm apart. Assume each copper atom has one extra electron.



Solution 6: As you can see the gravitational force acts to pull the two copper atoms together, while the electrostatic force acts to pull the atoms apart. The magnitude of the gravitational force and the electrical force are given as shown.

$$F_G = G\frac{mm}{r^2} \qquad\qquad F_E = k\frac{QQ}{r^2}$$

The ratio of these two forces is given as follows.

$$N = \frac{F_E}{F_G} = \frac{\left(\frac{kQQ}{r^2}\right)}{\left(\frac{Gmm}{r^2}\right)} = \frac{k}{G}\left(\frac{Q}{m}\right)^2$$

The mass of a single atom of copper is found using the molar mass of copper, (number of kg in a mole), and Avogadro's number, (number of atoms in a mole), as follows:

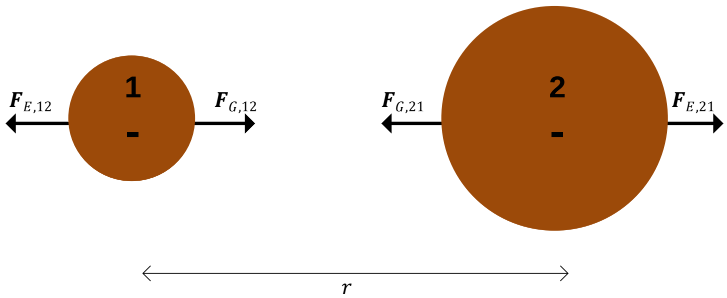
$$m = \frac{0.063546\ kg}{1\ mole\ of\ copper} \cdot \frac{1\ mole\ of\ copper}{6.022\ E^{23} atoms} = 1.055\ E^{-25} kg$$

Assuming each copper atom has an extra electron, the charge on each atom is, $Q = 1.6\ E^{-19}\ C$. The ratio is then given as

$$N = \frac{9E^9}{6.67\ E^{-11}}\left(\frac{1.6\ E^{-19}}{1.055\ E^{-25}}\right)^2$$
$$N = 3.1E^{32}$$

This tells us that the electrostatic force is *32 orders of magnitude* larger than the gravitational force!

For an alternate approach to compare these forces we start with a $1\ g$ solid metal sphere with a negative charge of $1\ C$. We then place this sphere a distance r from an identically charged solid metal sphere. Similar to the previous example, the electrostatic force will act to repel the two objects while the gravitational force will act to attract the two objects. We then ask: "How large does the second object needs to be in to order keep the objects in a state of equilibrium, i.e., $F_G = F_E$?



Equating the forces and solving for the larger mass, M, we have.

$$F_G = F_E$$
$$G\frac{mM}{r^2} = k\frac{QQ}{r^2}$$
$$M = \frac{k}{G}\left(\frac{Q^2}{m}\right)$$

And solving for M we have:

$$M = \frac{8.988\ E^9}{6.67\ E^{-11}}\left(\frac{1^2}{0.001}\right)$$
$$M = 1.35\ E^{23}\ kg$$

Which is only one order of magnitude less than the mass of the earth! Therefore, the second mass would need to be on the order of the mass of the entire earth to generate a gravitational force that is equal the electrostatic force that is supplied by 1 coulomb of charge!

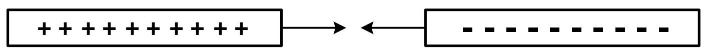
Final Summary for Electric Charge

Law of Electric Charged Objects

- Unlike charged objects attract each other
- Like charged objects repel each other

Like Charges Repel

Unlike Charges Attract



Material Types

Conductors: Materials in which some of the electrons are held loosely, “free electrons”, to the nucleus of the atoms and are free to move within the material from atom to atom.

Insulators: Materials in which there are no free electrons available to move within the material from atom to atom.

Ways to Charge an Object

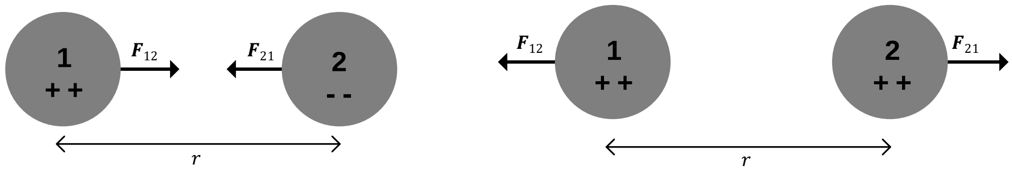
Friction: Electrons can be removed from one material and transferred to another by causing friction between the two (e.g., rubbing them together), making one of the objects negatively charged and the other positively charged.

Conduction: Occurs when a conducting charged object is brought into ***contact*** with a neutral conducting object. Electrons are transferred either to or from the charged object to the neutral object. The objects are then separated leaving the neutral object with a net charge.

Induction: Occurs when a conducting charged object is brought near a neutral conducting object, **without touching**. Electrons are either attracted or repelled *within* the neutral object towards the direction of the charged object, creating a polar neutral object. The polar object is then connected to a ground, (e.g., the earth). Electrons are then transferred to/from the ground from/to the polar object. After removing both the charged object and the ground connection the previously neutral object is charged.

Coulomb's Law

The electric force between two charged objects can be attractive or repulsive according to the rules from above.



The magnitude of the electric force is given by:

$$F_E = k\frac{Q_1 Q_2}{r^2}$$

Where, Q_1 is the magnitude of the charge on object 1, Q_2 is the magnitude of the charge on object 2, r is the distance between the objects, and k is a proportionality constant given by: $9E^9\ N \cdot m^2/C^2$. The unit of charge is the coulomb, C.

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