Physics 1 Mechanics – Potential Energy Diagrams

At the end of the previous lesson, we provided figures showing the total, kinetic, and potential energy as functions of time for an object in free-fall. In this lesson we specifically examine plots of potential energy versus distance. Much can be gained from these plots alone as we will show. Let's look at this idea assuming 1D motion, where we can write the following relationship for systems with only conservative forces.

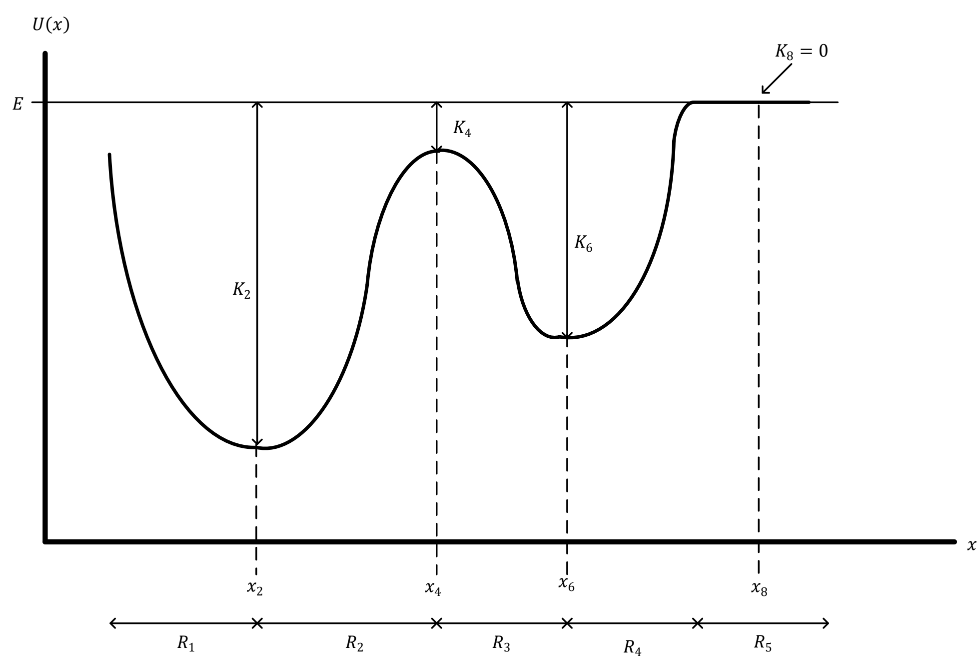
$$E = U(x) + K(x)$$

Recall the potential energy is related to the conservative force by

$$U(x) = -\int_0^x F(x)\,dx$$
$$F(x) = -\frac{dU(x)}{dx}$$

The second equation states that the force acting on an object at a point x, is equal to the negative slope of the potential energy curve at that same point. Based on this relationship we can gain a lot of information about the object just by looking at a potential energy curve.

We'll use the following example potential energy curve to see the things we can learn.



1. **Kinetic Energy:** If we are given a potential energy curve and the total energy, the kinetic energy at any position can be determined by rearranging the first equation above.

$$K(x) = E - U(x)$$

For example, on the diagram we can see at x_2, x_4, and x_6 the kinetic energy is K_2, K_4, and K_6 respectively. Furthermore at x_8 we can see that there is zero kinetic energy, $K_8 = 0$.

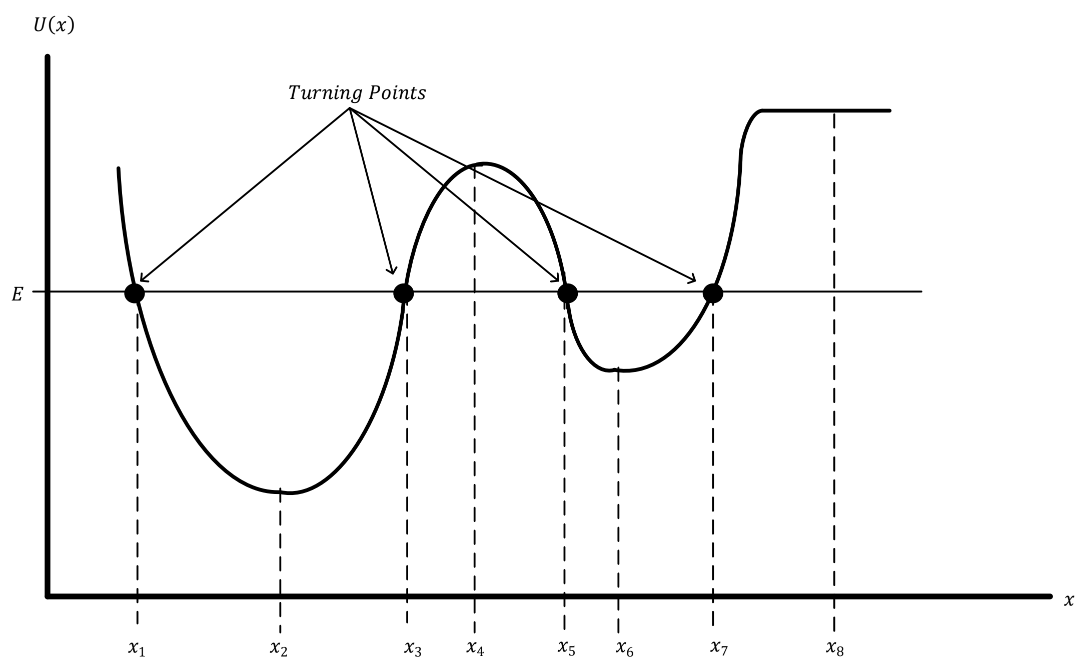
2. **Direction of the Force:** We stated above that the force is equal to the negative slope of the potential energy curve.

$$F(x) = -\frac{dU(x)}{dx}$$

Therefore, we can determine the direction of the force by simply observing the slope of the potential energy curve at different positions. Using the example curve above we can observe the direction of the force for different regions as follows:

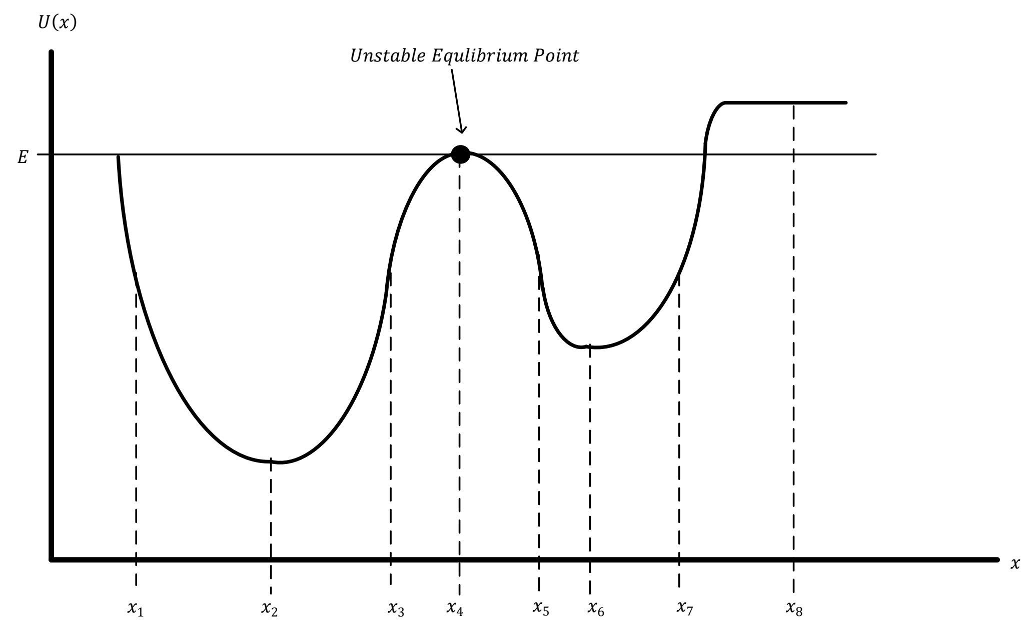
Region	1	2	3	4	5
Force Direction	+	-	+	-	0

3. **Turning Points:** To explain turning points let's start by assuming an object started with zero kinetic energy and a potential energy of E at x_1. Since we can never have more total energy than E and the kinetic energy is always positive, the potential energy can only change by moving to the right towards x_3. Similarly, if the object started at x_3 it can only move to the left back towards x_1. So, if an object with a total energy of E finds itself somewhere between x_1 and x_3 the object can only ever remain in this region since it is limited by the total energy (unless of course energy is added from outside the system). The points, x_1 and x_3, are called **turning points** since the object may oscillate between these two points but cannot move beyond them. Note we would have the same situation if the object started out somewhere between x_5 and x_7, where these points would now be the turning points.



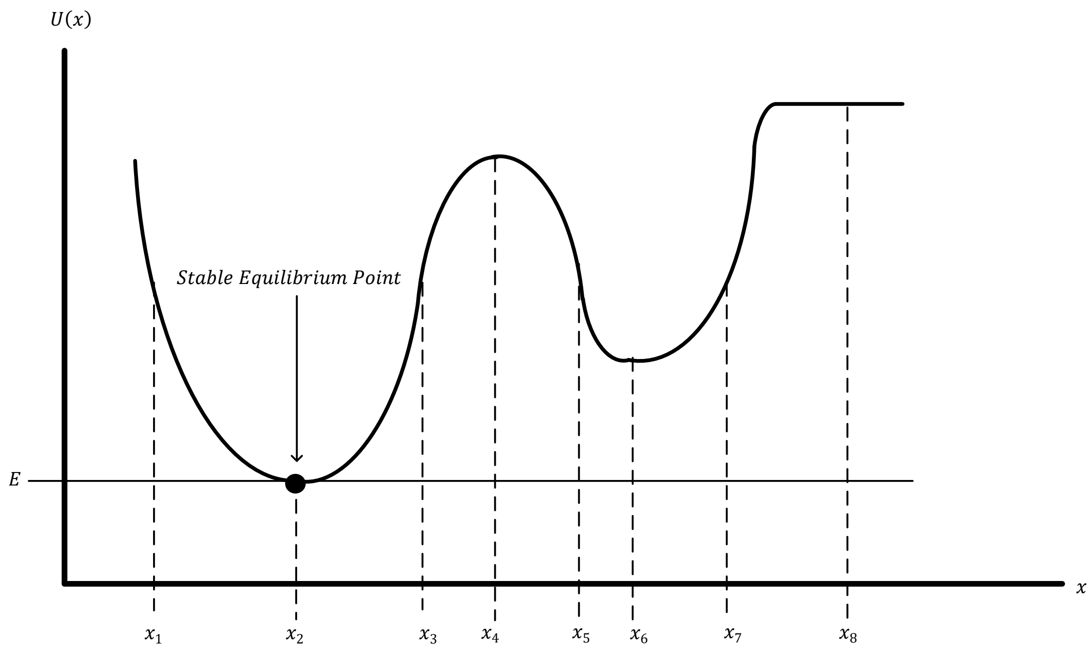
4. **Equilibrium Points:** Equilibrium points are points on the potential energy curve where the slope is equal to zero, hence the force is zero. There are however three different classifications for equilibrium points.

   1. **Unstable Equilibrium Points:** These occur at *local maximums* on the potential energy curve. In the figure below, suppose an object with a total energy of E found itself at x_4. Since the net force on this object is zero, we can assume the object is stationary (in equilibrium). However, if a very small force were to move the object slightly off this point the object would continue moving farther in the same direction. As an example, you can imagine a ball that is balanced on the top of a hemispherical surface. If a small force, say a wind force, were to move the object even slightly from this position it would continue to roll down the hemisphere. Hence this is point is considered **unstable.**



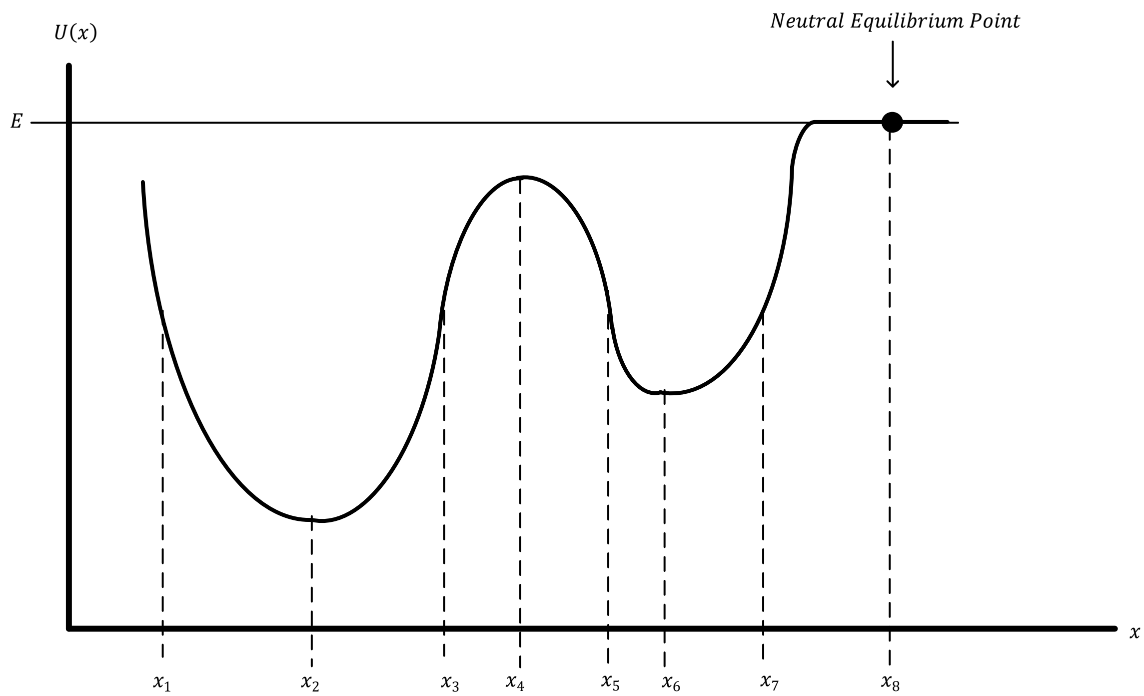
   2. **Stable Equilibrium Points:** These occur at *local minimums* on the potential energy curve**.** In the figure below, suppose an object with a total energy of E found itself at x_2. Since the net force on this object is zero, we can assume the object is stationary (in

equilibrium). In this case, if a small force were to move the object slightly off this point the object would quickly return to the equilibrium point due to the "restoring" conservative force. As an example, you can imagine a ball that is sitting at the bottom of a hemispherical bowl. If a small force, say a wind force, were to move the object slightly from this position the ball would quickly roll back to settle at the bottom of the bowl. Hence this is point is considered **stable.**



3. **Stable Equilibrium Points:** These occur at locations where the potential energy curve has a zero slope over a 'long' distance. In the figure below, suppose an object with a total energy of E found itself at x_8. Since the net force on this object is zero, we can assume the object is stationary (in equilibrium). In this case, if a small force were to

move the object slightly off this point the object would simply stay at this new location. As an example, you can imagine a ball that is sitting on a long flat surface. If a small force, say a wind force, were to move the object slightly from this position the ball simply remain at this new position. The point is neither stable nor unstable but is **neutral.**



Final Summary for Potential Energy Diagrams

Potential Energy

Assuming motion in one dimension where only conservative forces act we can write the following.

$$E = U(x) + K(x)$$

Kinetic Energy

The kinetic energy of an object can be determined from a potential energy diagram assuming we also know the total energy.

$$K(x) = E - U(x)$$

Direction of Conservative Force

The direction of the conservative force at any point can be determined by computing the slope of the potential energy curve at the desired point.

$$F(x) = -\frac{dU(x)}{dx}$$

Turning Points

Points on either side of a stable equilibrium point where the potential energy is equal to the total energy.

Equilibrium Points

- **Unstable:** Occur at local maximums of the potential energy curve.
- **Stable:** Occur at local minimums of the potential energy curve.
- **Neutral:** Occur at locations where the slope is zero over a substantial distance.

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