Physics 1 Mechanics – Energy

In the previous lesson we defined work and energy as shown.

Work: A number used to describe what is accomplished when a force acts on an object to move that object.

Energy: A number we associate with an object (or system of objects) to describe ***its ability to do work*** or to raise the temperature of the system.

In the previous lesson we focused on work. In this lesson we will focus on energy. The part of the energy definition we will focus on is its ***ability to do work.*** Energy comes in many forms such as chemical, electrical, nuclear, etc. However, we can classify each of these into one of two categories, which are summarized below.

- ***Kinetic Energy – Energy of Motion***
  - The energy an object possesses due to the motion of the object itself.
    - A moving train possesses kinetic energy because ***it has the ability to do work*** on a second object. For example, by hitting the second object with a force causing it to move over a certain distance. → $W = \boldsymbol{F} \cdot \boldsymbol{d}$

- ***Potential Energy – Energy of Position***
  - There are many types of potential energy, but we will focus on two:
    - *Gravitational Potential Energy*
      - An object being held at a certain height above the ground has potential energy because ***it has the ability to do work*** on a second object if it is dropped. For example, when it eventually comes into contact with a second object it will cause that object to move over a certain distance. Note, the higher above the ground the more potential energy the object has.
    - *Elastic Potential Energy*
      - A spring that is held in either a stretched or compressed position has potential energy because ***it has the ability to do work*** on a second object. For example, if the spring is released it will begin to move back to its equilibrium position. If it comes into contact with another object it will cause that object to move over a distance. Note, the more stretched or compressed the more potential energy the spring has.

We begin this lesson with kinetic energy and its relationship to work.

Kinetic Energy – Energy of Motion

We can derive an expression for the kinetic energy of an object using the following 1D scenario. When an object moves over a distance, d, as a result of a net force, F_{net}, we know from the previous lesson that the net work done on that object is given as

$$W_{net} = F_{net}d$$

Applying Newton's 2nd law to the same object we can also write the following.

$$F_{net} = ma$$

If we then assume the acceleration is constant, we can use kinematics to express the acceleration as follows.

$$v_f^{\,2} - v_i^{\,2} = 2ad$$
$$a = \frac{v_f^{\,2} - v_i^{\,2}}{2d}$$

Substituting into the first equation, we can find an alternate expression for the work as shown.

$$W_{net} = F_{net}d$$
$$W_{net} = mad$$
$$W_{net} = m\left(\frac{v_f^{\,2} - v_i^{\,2}}{2d}\right)d$$
$$W_{net} = \frac{m\left(v_f^{\,2} - v_i^{\,2}\right)}{2}$$
$$W_{net} = \left(\frac{1}{2}mv_f^{\,2}\right) - \left(\frac{1}{2}mv_i^{\,2}\right)$$

As you can see the work done on the object is a function of the initial and final state of the velocity and mass of the object. This measure of the state of the object, $\frac{1}{2}mv^2$, we refer to as its kinetic energy. With this we can write the following so called ***Work-Kinetic Energy Principle***.

$$W_{net} = K_f - K_i$$
$$W_{net} = \Delta K$$

We see now that the work done on an object causes a change its kinetic energy. More specifically, if an object is made to increase its speed, i.e., increase its kinetic energy, then positive work is done on that object, and if an object is made to decrease its speed, i.e., decrease its kinetic energy, then negative work is done on that object. We summarize below.

The Kinetic Energy of an object is given by: $K = \frac{1}{2}mv^2$
Work-Kinetic Energy Principle $W_{net} = \Delta K$ $W_{net} = K_f - K_i$

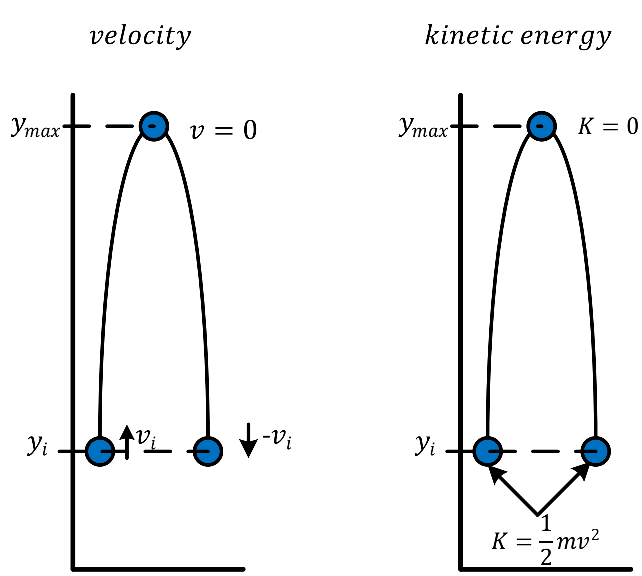
Potential Energy – Energy of Position

In this lesson we focus on two types of potential energy, gravitational and elastic. For gravitational potential energy we stated that "*an object being held at a certain height above the ground has potential energy*". For elastic potential energy we stated that "*A spring that is held in either a stretched or compressed position has potential energy*". A more formal definition of potential energy is given below.

Potential Energy
The energy associated with the arrangement of a system of objects that exert conservative forces on each other.

In the gravitational case we have the arrangement of an object and the earth. In the elastic case the system is the spring itself and its position relative to its equilibrium position. To make these concepts more concrete let's develop the idea of both potential energy and a conservative force using the gravitational example below.

Consider a ball being thrown straight up in the air with an initial velocity of v_i from an initial height of y_i. We know from our review of kinetic energy that the ball has an initial kinetic energy, $K_i = \frac{1}{2}mv_i^2$. Furthermore, we know from experience that the balls velocity will steadily decrease until it reaches a maximum height with zero velocity. At this point the ball can be said to have zero velocity and therefore zero kinetic energy. The ball will then begin to fall back to the earth with an increasing velocity, and therefore increasing kinetic energy. When the ball returns to the initial height, y_i, its velocity will have the same magnitude it started with, and therefore the same kinetic energy. These observations can be proven using kinematic analysis from previous lessons. However, in terms of energy we can now ask: "Where does the energy go on the way up and how did the ball gain all the energy back when it fell back to its starting position?".



We can answer this question using the concept of potential energy as follows.

1. As the ball rises the *kinetic energy* of the ball-earth system is transferred into *potential energy* stored in the arrangement system, where the force of gravity, a *conservative force*, is acting.
2. As the ball falls the *potential energy* of the ball-earth system is transferred back to *kinetic energy*.

Since the work being done on the ball is solely from the gravitational force, we can use the work-kinetic energy principle from above to write the following.

$$W_{net} = W_G = \Delta K$$

Using the initial position of the ball, y_i, and the final position, y_{max}, we find that as the ball rises gravity does negative work.

$$W_{G\_up} = K_f - K_i = 0 - K_i = -\frac{1}{2}mv_i^{\,2}$$

Conversely, as the ball falls gravity does positive work.

$$W_{G\_down} = K_f - K_i = K_f - 0 = \frac{1}{2}mv_i^{\,2}$$

The net work done on the ball by gravity then is

$$W_{net} = W_{G\_up} + W_{G\_down} = 0$$

Which leads us to a fundamental statement about a conservative force.

Conservative Force
When the net work done by a force to move an object in a closed path is zero, the force is *conservative*.

It's important to understand that potential energy applies ***only*** to conservative forces. A force that is not conservative is called *non-conservative*. An example of a non-conservative force is the friction force. When a block slides along a rough surface, the kinetic energy is transferred to thermal energy (heat). This energy is effectively lost and cannot be fully recovered and made to return to kinetic energy of the block as it did with the ball under the conservative gravitational force.

Back to the example, we see that a change in kinetic energy is matched by negative change in the potential energy. Using the notation, U, for potential energy we can write the following fundamental relationship for systems that are under the influence of conservative forces only.

$$\Delta U = -\Delta K$$

Substituting $\Delta K = W_c$ using the work-kinetic energy principle we find.

$$\Delta U = -W_c$$

Where, the subscript W_c represents the work done by conservative forces.

With this, we can now derive expressions for both gravitational and elastic potential energy.

Gravitational Potential Energy: In the previous lesson we found the work done by gravity when an object changes its height by Δy is given as:

$$W_G = -mg\Delta y$$

And since gravity is a conservative force we can write:

$$\Delta U = -W_G$$
$$\Delta U = -(-mg\Delta y)$$
$$\Delta U = mg\Delta y$$

And if we let $U(y_i = 0) = 0$, we can define the gravitational potential energy as a function of y.

$$U(y) = mgy$$

Elastic Potential Energy: We similarly found the work done by the spring force when the spring is stretched or compressed from its equilibrium position by Δx is given as:

$$W_s = -\frac{1}{2}k(\Delta x)^2$$

And in the same way as we did for gravity, we can define the following for the elastic potential energy.

$$\Delta U = \frac{1}{2}k(\Delta x)^2$$

$$U(x) = \frac{1}{2}kx^2$$

We can also derive general expressions for the relationship between conservative forces and potential energy. Using the integral equation for the work done by a variable 1D conservative force, along with $\Delta U = -W_c$, we can write the following.

$$W_c = \int_{x_1}^{x_2} F_c(x)dx$$
$$\Delta U = -\int_{x_1}^{x_2} F_c(x)dx$$

Next, if we let $U(x_1 = 0) = 0$, we can write the potential energy as a function of x.

$$U(x) = -\int_0^x F_c(\tau)d\tau$$

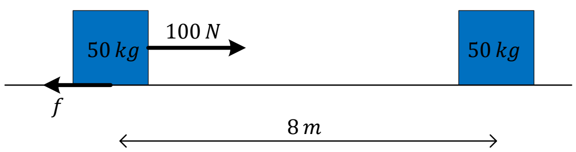
Finally, we can differentiate both sides and also write the following.

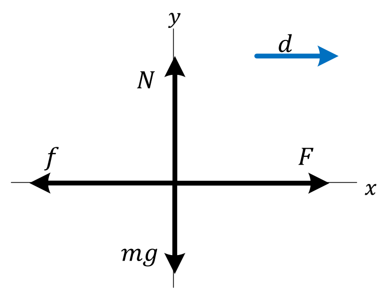
$$F_c(x) = -\frac{dU}{dx}$$

Example 1: A 50 kg box initially at rest is pushed 8 m along a rough, horizontal floor with a constant applied force of 100 N. The coefficient of friction between the box and the floor is 0.2. Find the following values:

a) The work done by the applied force.
b) The work done by the friction force.
c) The change in kinetic energy of the box.
d) The final speed of the box.

Solution 1: A sketch of the scenario, along with a free-body diagram is shown below.





1a. The work done by the applied force is given as

$$W_F = Fd = 100 \cdot 8 = 800\,J$$

1b. The friction force, $f = uN = umg$, is applied in the opposite direction of motion and therefore does negative work.

$$W_f = -fd = -umgd = -0.2 \cdot 50 \cdot 9.8 \cdot 8 = -784\,J$$

1c. We can use the work-kinetic energy principle to find the change in kinetic energy. Since the y directed forces are not doing work, the net work is the sum of the work from the applied force and the friction force.

$$\Delta K = W_{net}$$
$$\Delta K = W_F + W_f$$
$$\Delta K = (800) + (-784) = 16\,J$$

1d. Since the box is initially at rest, $K_i = 0$, and therefore, $\Delta K = K_f$. The speed is then found as follows.

$$K_f = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2K_f}{m}}$$
$$v = \sqrt{\frac{2 \cdot 16}{50}} = 0.8\,m/s$$

Example 2: A $80\ kg$ hiker sets out in the morning to climb a mountain. After climbing to an elevation of 600 meters the climber rests to have lunch. In the afternoon, the hiker climbs another 300 meters. a.) How much did the climbers gravitational potential energy change in the afternoon climb? b.) What is the hikers gravitational potential energy at the end of the climb?

Solution 2a.) The change in gravity potential energy is given as

$$\Delta U_G = mg\Delta h$$

Where Δh is the vertical change in distance. Note the horizontal distance traveled by the hiker does not contribute to the hikers gravitational potential energy.

$$\Delta U_G = 80 \cdot 9.8 \cdot 300 = 2.352E^5\ J$$

The hiker gained $2.352E^5\ J$ of gravitational potential energy in the afternoon climb.

2b.) To find the absolute potential energy we need to choose a reference position for zero potential energy. Although this point is purely arbitrary, it makes the most sense to take the base of the mountain as zero gravitational potential energy. Therefore, the potential energy of the climber at the end of the journey is given as follow.

$$U(900) = mgh = 80 \cdot 9.8 \cdot 900 = 7.056E^5\ J$$

Example 3: A single conservative force

$$F(x) = 3x + 6$$

acts on a $6\ kg$ particle, where x is in meters. As the particle moves from $x_1 = 1$ to $x_2 = 6$, calculate the change in potential energy of the particle.

Solution 3: As the force is variable and conservative, we can use the general integral we developed above to find the change in potential energy of the particle.

$$\Delta U = -\int_{x_1}^{x_2} F_c(x)dx$$

$$\Delta U = -\int_{1}^{6} (3x + 6)dx$$

$$\Delta U = -\left(\frac{3}{2}x^2 + 6x\right)_1^6$$

$$\Delta U = -\left(\left(\frac{3}{2}6^2 + 6 \cdot 6\right) - \left(\frac{3}{2}1^2 + 6 \cdot 1\right)\right)$$

$$\Delta U = -82.5\ J$$

Final Summary for Energy

- ***Kinetic Energy – Energy of Motion***
  - The energy an object possesses due to the motion of the object itself.
    - A moving train possesses kinetic energy because ***it has the ability to do work*** on a second object. For example, by hitting the second object with a force causing it to move over a certain distance. → $W = \mathbf{F} \cdot \mathbf{d}$

- ***Potential Energy – Energy of Position***
  - *Gravitational Potential Energy*
    - An object being held at a certain height above the ground has potential energy because ***it has the ability to do work,*** e.g., on a second object if it is dropped.
  - *Elastic Potential Energy*
    - A spring that is held in either a stretched or compressed position has potential energy because ***it has the ability to do work*** e.g., to move a second object.

Kinetic Energy

The Kinetic Energy of an object is given by	*Work-Kinetic Energy Principle*
$K = \frac{1}{2}mv^2$	$W_{net} = \Delta K$ $W_{net} = K_f - K_i$

Potential Energy

Conservative Force - When the net work done by a force to move an object in a closed path is zero, the force is *conservative*. Potential Energy is associated with conservative force **only**.

The Change in Potential Energy is given by:

$$\Delta U = -\Delta K$$
$$\Delta U = -W_c$$

Gravitational Potential Energy

$$\Delta U = mg\Delta y$$

If we let $U(y_i = 0) = 0$, we can define the gravitational potential energy as a function of y as shown.

$$U(y) = mgy$$

Elastic Potential Energy

$$\Delta U = \frac{1}{2}k(\Delta x)^2$$

If we let $U(x_i = 0) = 0$, we can define the elastic potential energy as a function of x as shown.

$$U(x) = \frac{1}{2}kx^2$$

General relationship between Potential Energy and an associated Conservative Force

$U(x) = -\int_0^x F_c(\tau)d\tau$	$F_c(x) = -\frac{dU}{dx}$

By: ferrantetutoring