Physics 1 Mechanics - Mass and Weight

Mass vs Weight

Mass and weight are properties of an object that in everyday life are sometimes *mistakenly* thought to be identical. Let's examine these properties more closely to precisely understand the differences.

Mass:

Mass can be thought of as a measure of the "quantity of matter" an object contains. However, in physics, it is more useful to define mass through Newton's second law, which you'll recall can be written as follows.

$$a = \frac{\sum F}{m}$$

From this relationship, we can see that the acceleration of an object is directly proportional to the net force on an object and inversely proportional to the mass of the object. Let's illustrate with two examples:

1. Apply a $10\ N$ force to an object with $1\ kg$ mass.
   - The acceleration of that object would be $10\ m/s^2$
2. Apply the same $10\ N$ force to an object with a $10\ kg$ mass.
   - The acceleration of that object would be $1\ m/s^2$

With this we can define the mass of an object as: "A *measure of an objects ability to resist being accelerated by a force"*. We call this the inertia of a body, and sometimes refer to this definition as the inertial mass definition.

Weight:

Galileo first postulated that all objects dropped near the surface of the earth fall to the earth with the same acceleration, which we call the acceleration of gravity, g. From Newton's first law we know if an object is accelerating there must be a net force on that object. The force that is causing a falling object to accelerate is from the earth and is known as the gravitational force, F_g. From Newton's second law, (and ignoring other forces such as wind, etc.), we know that the gravitational force on this object is.

$$\sum F = ma$$

$$F_g = mg$$

This if the force we commonly associate with the weight of an object, W.

$$W = F_g$$

Note however, that this force is specifically defined for objects near the surface of the *earth*. You can imagine then that if we were to drop an object on the surface of the moon or some other planet, the object would fall with an acceleration that is different from g. Therefore, it would be more precise to say: “The force of gravity due to the *earth*”, and “The weight of an object on *earth*”.

$$W_E = F_{gE} = mg_E$$

In general, the weight of an object on planet, P, can be given as

$$W_P = F_{pM} = mg_p$$

Where g_p is the acceleration caused by the planet, which presumably would be different than that of the earth.

In summary, two key observation about mass and weight can be stated as follows.

- **The mass of an object is the same regardless of where the object is located in space.**
- **The weight of an object will change based on where that object is located in space.**

Let’s explore these concepts with two example problems.

Example 1: (Weight on Earth versus Moon)

A person weighs 700 Newtons on earth.

a.) What is the person’s mass on earth?
b.) What is the person’s mass and weight on the surface of the moon?

Solution: Part a.

We will formulate the gravitation force in more detail in a later lesson. For now, we simply provide a formula for the acceleration due to gravity for an object near the surface of a planet.

$$g_p = \frac{GM_p}{(r_p)^2}$$

Where, G is the Gravitational constant: $6.67E^{-11}\ N \cdot m^2/kg^2$, M_p is the mass of the planet in kilograms, and r_p is the radius of the planet in meters.

For earth, we are familiar with $g_E = 9.8\ m/s^2$. Using the formula with the mass and radius of the earth we can show this computation below.

$$g_E = \frac{6.67E^{-11} * 5.98E^{24}}{(6.38E^6)^2}$$
$$g_E = \frac{6.67 * 5.98E^1}{(6.38)^2}$$
$$g_E \cong 9.8\ m/s^2$$

The mass of the person can then be computed as shown.

$$m = \frac{W_E}{g_E}$$
$$m = \frac{700}{9.8}$$
$$m = 71.43\ kg$$

Solution Part b. Since the mass of the person *does not change* based on their location, the mass of the person on the moon is also $71.43\ kg$. However, weight *does change*, and it is a function of both the mass and the radius of the planet. The mass of the moon is $M_m = 7.35E^{22}\ kg$, and the radius is $r_m = 1.74E^6\ m$. Therefore, the acceleration of an object near the surface of the moon is more than five times less than that of the earth.

$$g_m = \frac{GM_m}{(r_m)^2}$$
$$g_m = \frac{6.67E^{-11} * 7.35E^{22}}{(1.64E^6)^2}$$
$$g_m \cong 1.8\ m/s^2$$

The weight of the person on the moon is then found as follows.

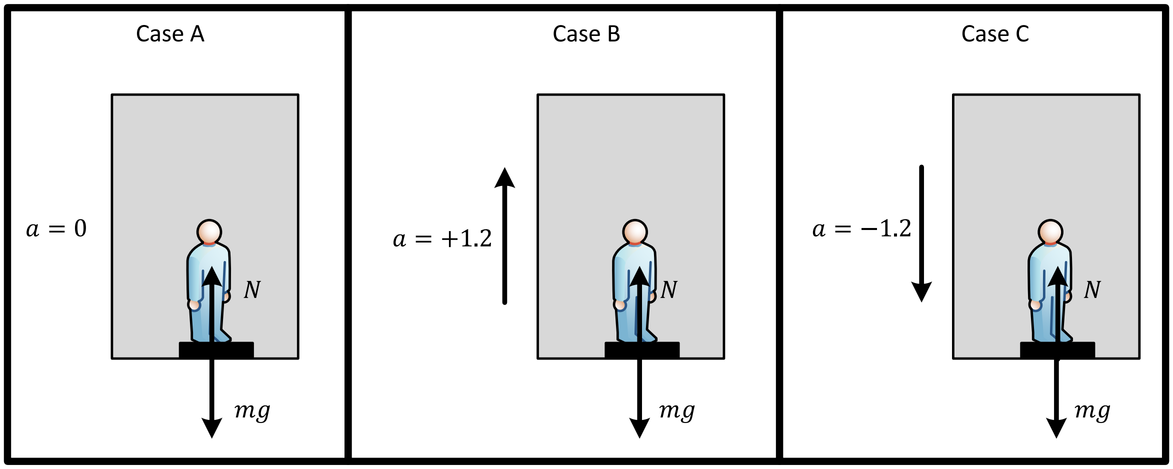
$$W_m = mg_m$$
$$W_m = 71.43 \cdot 1.8$$
$$W_m = 71.43\left(\frac{6.67E^{-11} * 7.35E^{22}}{(1.64E^6)^2}\right)$$
$$W_m = 130.2\ N$$

Example 2: (Weight in an elevator)

We commonly measure our weight on earth on a typical bathroom scale when we are not moving relative to the earth. Determine the *perceived weight* of a 72 kg person if we placed the scale in an elevator in the following three scenarios.

A. The acceleration of the elevator is zero. (Note this can be due to $v = 0$ or $v = C$).
B. The elevator is accelerating downward at a rate of 1.2 m/s^2.
C. The elevator is accelerating upward at a rate of 1.2 m/s^2.

Solution: The three cases are shown below.



A persons weight is indicated by the reading on the scale and is a result of the upward normal force, N. Therefore, we can find the weight by writing down Newton's second law and solving for the normal force. In case A, the acceleration is zero and therefore the two forces exactly balance. However, this is not the case in the next two scenarios, as shown below.

Case A	***Case B***	***Case C***
$\sum F = 0$ $N = mg$ $N = 72 * 9.8$ $\boldsymbol{N = 705.6\ N}$	$\sum F = ma$ $N - mg = ma$ $N = m(g + a)$ $N = 72(9.8 + 1.2)$ $\boldsymbol{N = 792\ N}$	$\sum F = ma$ $N - mg = m(-a)$ $N = m(g - a)$ $N = 72(9.8 - 1.2)$ $\boldsymbol{N = 619.2\ N}$

- **Case A**: The two forces are equal, and the scale indicates the weight we would expect.
- **Case B/C**: Since the elevator is accelerating, the two forces do not sum to zero and the scale indicates a different value than what is expected.
  - **Case B**: The elevator is accelerating upward, and the scale indicates a bigger value than expected, i.e., your perceived weight increases.
  - **Case C**: The elevator is accelerating downward, and the scale indicates a smaller value than expected, i.e., your perceived weight decreases.

Note, when the downward acceleration is equal to g, we are in what is referred to as *free fall*, and we experience weightlessness!

$$\sum F = ma$$
$$N - mg = m(-g)$$
$$N = m(g - g)$$
$$N = 72(0)$$
$$N = 0\ N$$

Final Summary for Weight vs. Mass

Mass

Definition 1: A measure of the "quantity of matter" an object contains.
Definition 2: A measure of an objects ability to resist acceleration by a force (inertial mass) .

The mass of an object is the same regardless of where the object is located in space.

Weight

Definition: The gravitational force on an object due to another body, commonly the earth.

The weight of an object will change based on where that object is located in space.

The weight of an object of mass m on a plant P is given by.

$$W_P = F_{gP} = mg_P$$

With

$$g_P = \frac{GM_P}{(r_P)^2}$$

Where, G is the Gravitational constant: $6.67E^{-11}\, N \cdot m^2/kg^2$, M_p is the mass of the planet in kilograms, and r_p is the radius of the planet in meters.

For planet earth we have

$$g_E = \frac{6.67E^{-11} * 5.98E^{24}}{(6.38E^6)^2}$$
$$g_E = \frac{6.67 * 5.98E^1}{(6.38)^2}$$
$$g_E \cong 9.8\ m/s^2$$

Perceived Weight

When a person is accelerating near the surface of a planet, their perceived weight will change.

- For upward acceleration, your perceived weight increases.
- For downward acceleration, your perceived weight decreases.

Note, when the downward acceleration is equal to g, we are in what is referred to as *free fall*, and we experience weightlessness!

$$\sum F = ma$$
$$N - mg = m(-g)$$
$$N = m(g - g)$$
$$N = 0$$

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