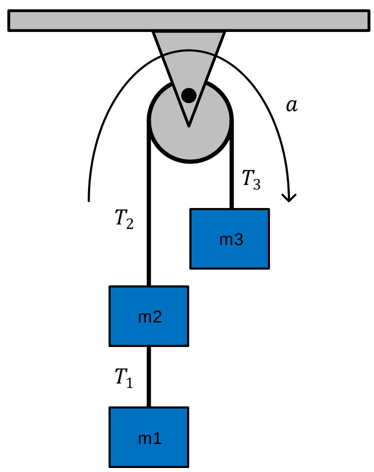
Physics 1 Mechanics - Pulleys and Inclines

In the previous lesson we started using Newton's laws to solve some basic dynamics problems. In this lesson we continue this process, using slightly more complex systems. The dynamic systems we will analyze include masses connected to pulleys and masses on inclined surfaces. In this lesson we assume all pulleys are massless and frictionless. This assumption translates into the pulleys acting to change the direction of motion only. Therefore, the tension in the rope is the same on each side of a pulley. In later lessons we will introduce pulleys that are not massless, and hence contain a type of inertia that resists being rotated. Lastly, we assume all inclined surfaces to be completely frictionless, and hence offer no resistance to motion. We follow the same guideline from the previous lesson to solve these more complex dynamics problems. Let's begin with the examples.

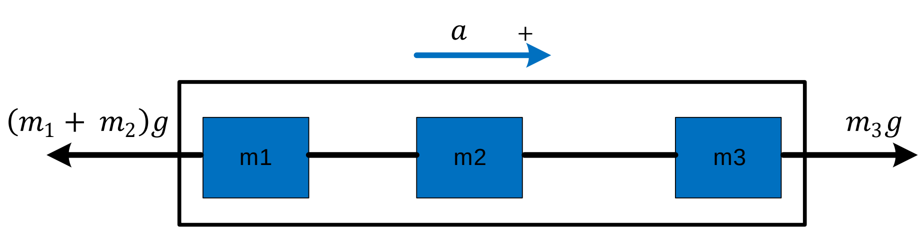
Example 1: (Single Pulley)
A pulley system is arranged as shown, with $m_1 = 2$, $m_2 = 3$, and $m_3 = 10$. The pulley is considered massless and frictionless. Find the acceleration of the system and the tension in the strings.



Solution 1: First a few observations to help us solve the problem.

- Since the pulley is massless and frictionless it acts only to change the direction of motion and the magnitude of T_2 is equal to T_3.
- Since m_3 is greater than $(m_1 + m_2)$, we assume the system accelerates in the direction shown in the sketch.

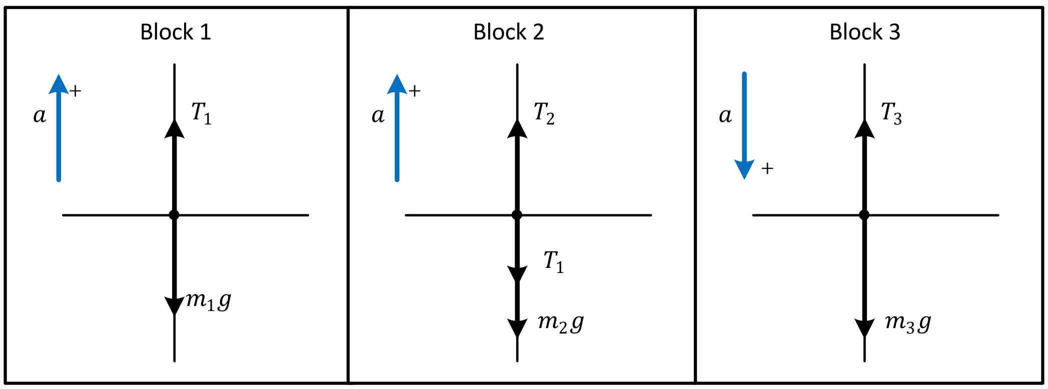
Based on the first observation we may treat this problem just like we did with compound bodies from the last section and find the acceleration by considering the system as a whole. A free-body diagram removing the pulley is shown below.



Next, we write Newton's 2nd law for all three masses combined and solve for the acceleration.

$$\sum F_x = m_T a$$
$$m_3 g - (m_1 + m_2)g = (m_1 + m_2 + m_3)a$$
$$a = \frac{m_3 g - (m_1 + m_2)g}{(m_1 + m_2 + m_3)}$$
$$a = \frac{10 * 9.8 - (2 + 3)9.8}{(2 + 3 + 10)}$$
$$a = 3.27\ m/s^2$$

To find the tensions, however, we need to analyze the blocks individually. Note the direction of positive acceleration in each free-body diagram.



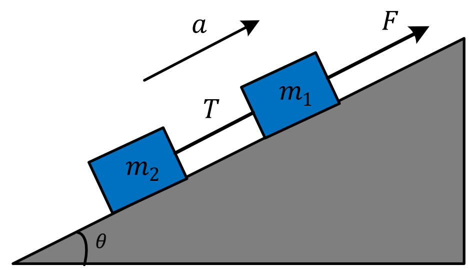
Block 1	***Block 2***
$\sum F_y = m_1 a$ $T_1 - m_1 g = m_1 a$ $T_1 = m_1(g + a)$ $T_1 = 2(9.8 + 3.27)$ $T_1 = 26.14\ N$	$\sum F_y = m_2 a$ $T_2 - T_1 - m_2 g = m_2 a$ $T_2 = T_1 + m_2(g + a)$ $T_2 = 26.14 + 3(9.8 + 3.27)$ $T_2 = 65.35\ N$

For block 3 we know that $T_3 = T_2$. We can use this to confirm the acceleration already computed above.

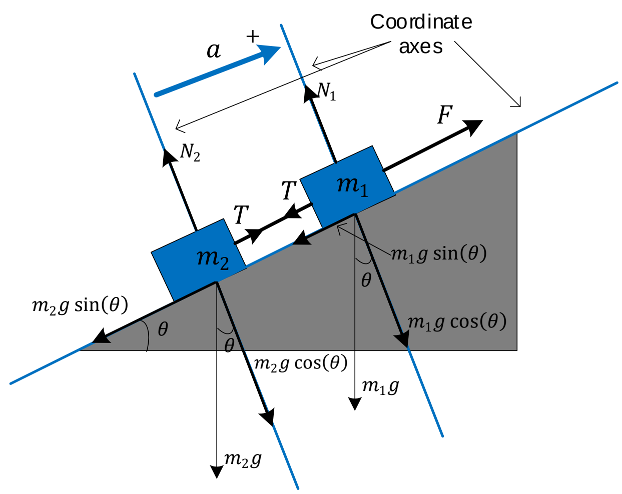
Block 3
$\sum F_y = m_3 a$ $m_3 g - T_3 = m_3 a$ $a = \frac{m_3 g - T_3}{m_3}$ $a = \frac{10 * 9.8 - 65.35}{10}$ $a = 3.27\ m/s^2$

Example 2: (Inclined Plane)

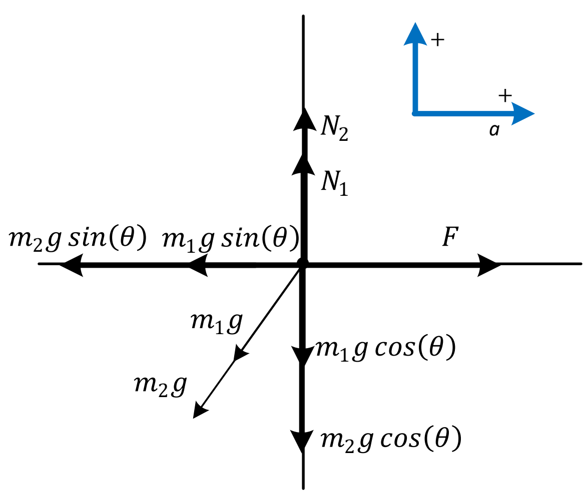
Two connected blocks of $m_1 = 25\ kg$, and $m_2 = 45\ kg$ are being pulled up a frictionless inclined plane with an acceleration of 4 m/s^2. What is pulling force? What is the tension in the cord pulling on the lower block?



Solution 2: For inclined planes it's usually best to create an axis along the incline. Below we redraw the sketch with the axis superimposed. The forces are also shown in the sketch.



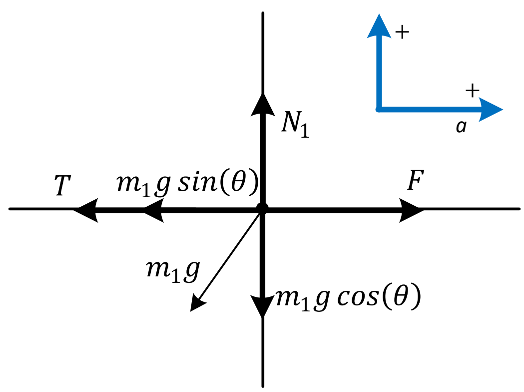
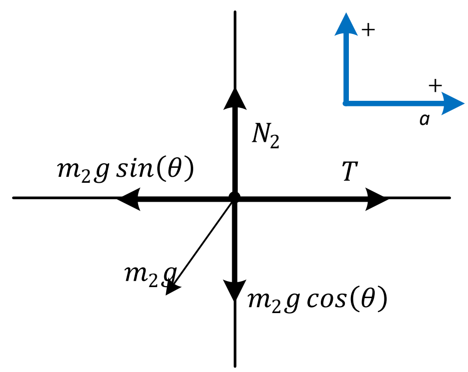
In this case we can again combine the objects since the two tensions will cancel as in previous problems. The free-body diagram for the combined object is shown below.



To solve for the force, F, we can use Newton's 2nd law in the x direction as follows.

$$\sum F_x = (m_1 + m_2)a$$
$$F - m_1 g\,sin(\theta) - m_2 g\,sin(\theta) = (m_1 + m_2)a$$
$$F = (m_1 + m_2)a + (m_1 + m_2)g\,sin(\theta)$$
$$F = (m_1 + m_2)(a + g\,sin(\theta))$$
$$F = (25 + 45)(4 + 9.8\,sin(20^{\circ}))$$
$$F = 514.63\ N$$

To find the tension in the rope we need to analyze either of the blocks separately. We'll solve for T using both blocks for illustration. The free-body diagrams for each are shown below.

Block 1	***Block 2***
	

Block 1:

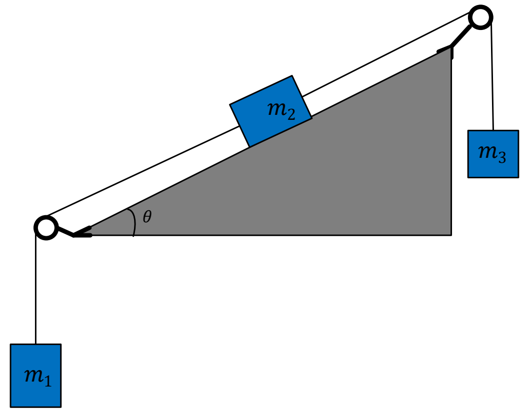
$$\sum F_x = m_1 a$$
$$F - T - m_1 g\,sin(\theta) = m_1 a$$
$$T = F - m_1 g\,sin(\theta) - m_1 a$$
$$T = F - m_1(g sin(\theta) + a)$$
$$T = 514.63 - 25(9.8\,sin(20^{\circ}) + 4)$$
$$T = 330.84\ N$$

Block 2:

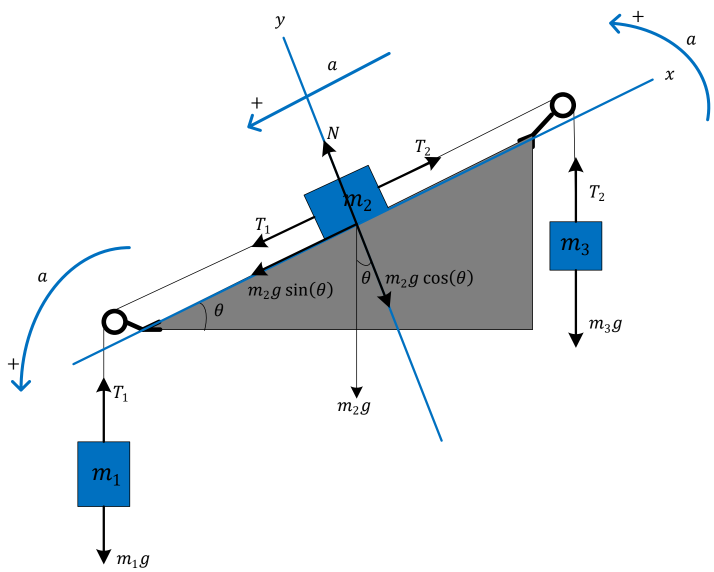
$$\sum F_x = m_2 a$$
$$T - m_2 g\,sin(\theta) = m_2 a$$
$$T = m_2 g\,sin(\theta) + m_2 a$$
$$T = m_2(g sin(\theta) + a)$$
$$T = 45(9.8\,sin(20^{\circ}) + 4)$$
$$T = 330.84\ N$$

Example 3: (Pulley and Inclined Plane)

A system consisting of two pulleys, three blocks, and an inclined plane are configured as shown in the figure below. The ramp makes an angle of 25° with the horizontal and the mass of block 1, 2, and 3 are 10, 3 and 2 kg respectively. The pulleys are massless and frictionless as is the surface of the incline. Find the acceleration of the system and the tension in the rope.

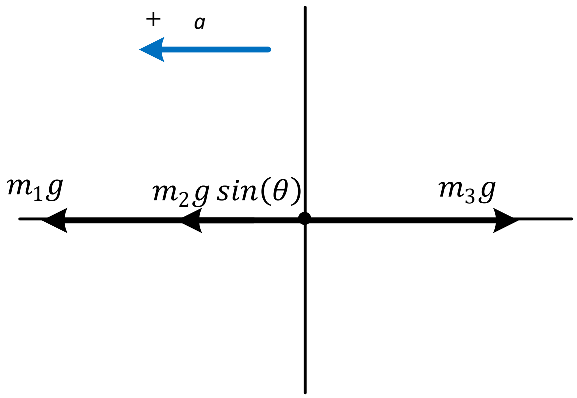


Solution 3: As in the previous problem we redraw the sketch with a tilted axis for block 2.



Note that we keep the acceleration positive along the entire system. This make things clearer when combining blocks to solve the system.

Similar to example 1, the pulleys in this case act to change the direction of motion only. Therefore, the tension on opposite sides of the pulleys will have the same magnitude and we can combine the blocks to solve for the acceleration of the system as we did previously. The free-body diagram for the combined blocks is shown below. We show the forces in the direction of the acceleration only.



We can use Newton's 2nd law to solve for acceleration as shown below.

$$\sum F_x = (m_1 + m_2 + m_3)a$$

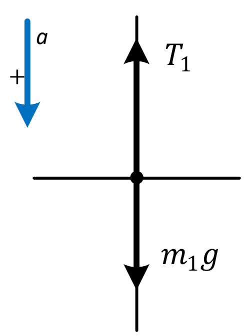
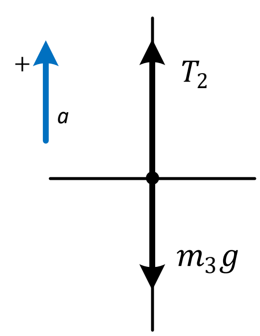
$$m_1g + m_2g\,sin(\theta) - m_3g = (m_1 + m_2 + m_3)a$$

$$a = \frac{g(m_1 + m_2\,sin(\theta) - m_3)}{(m_1 + m_2 + m_3)}$$

$$a = \frac{9.8(10 + 3\,sin(25^{\circ}) - 2)}{(10 + 3 + 2)}$$

$$a = 6.1\ m/s^2$$

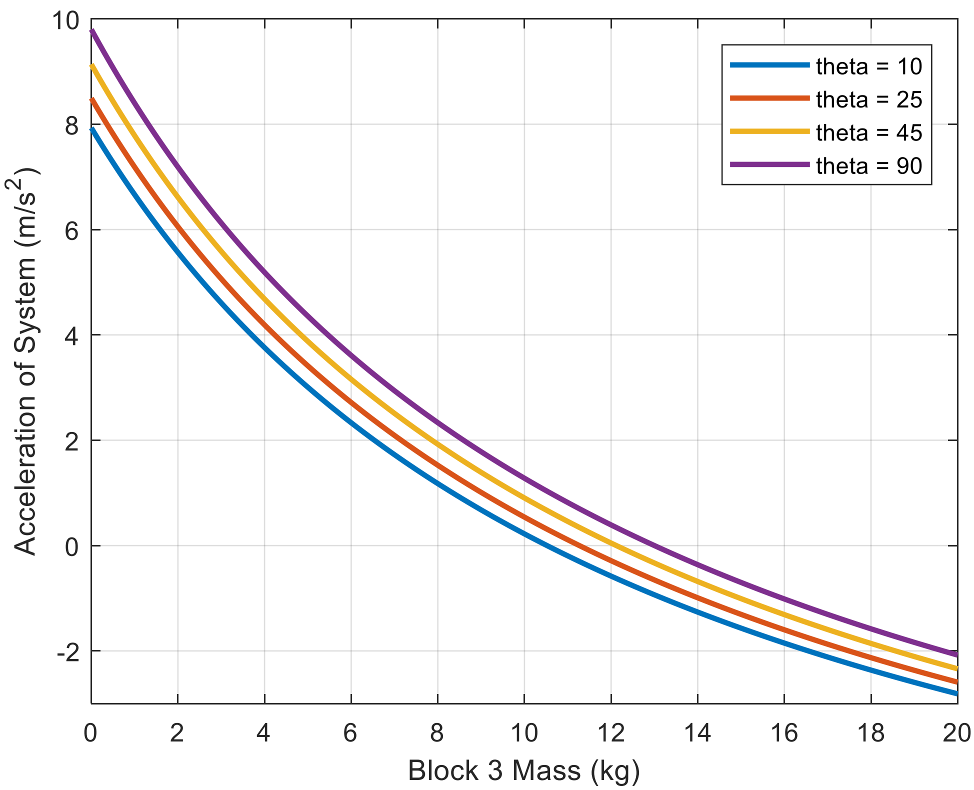
Next, we need to look at the blocks separately to solve for the tensions, T_1 and T_2. Since we already solved for the acceleration, we can use the 1st and 3rd block to easily solve for T_1 and T_2, respectively.

Block 1	***Block 3***
	
$\sum F_y = m_1a$ $m_1g - T_1 = m_1a$ $T_1 = m_1(g - a)$ $T_1 = 10(9.8 - 6.1)$ $T_1 = 37\ N$	$\sum F_y = m_3a$ $T_2 - m_3g = m_3a$ $T_2 = m_3(g + a)$ $T_2 = 2(9.8 + 6.1)$ $T_2 = 31.8\ N$

Lastly, for additional insight let's look a bit closer at the example 3. The system is accelerating to the left because the summation of the forces pulling to the left is greater than the summation of the forces pulling to the right. The equation we derived for the acceleration of the system is shown below.

$$a = \frac{g(m_1 + m_2\,sin(\theta) - m_3)}{(m_1 + m_2 + m_3)}$$

We can use this equation to see how the acceleration of the system responds when letting m_3 vary for different angles of incline. Below we plot the acceleration for angles, 10°, 25°, 45°, and 90°, each time varying m_3 from 0 to $20\ kg$.



Key Observations:

- For all curves, i.e., angles of inclination, there is a point when m_3 becomes just large enough so that the forces balance, resulting in an acceleration of zero. For larger values of m_3 the system begins to accelerate in the opposite direction.
- As the angle of inclination gets larger the curves are shifted up.
- The maximum acceleration for each curve is when $m_3 = 0$
    - When $m_3 = 0$ and the angle is 90°, the remaining blocks experience free fall with an acceleration of $9.8\ m/s^2$.

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