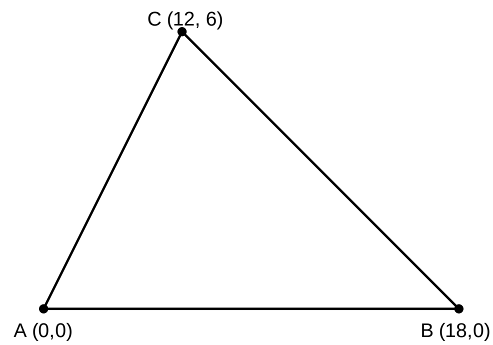
Median, Altitude, and Perpendicular Bisector of a Triangle Example

Find all three median lines, altitude lines, and perpendicular bisector lines for the triangle shown to the right



Median Lines:

Recall the median lines go from one of the vertices to the midpoint of the opposite side. Therefore, we can start by finding the midpoint of each segment of the triangle. Recall the midpoint between two points, (x_1, y_1) and (x_2, y_2) is given as:

$$(x_{mp}, y_{mp}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$(x_{mp\_AB}, y_{mp\_AB}) = \left(\frac{0+18}{2}, \frac{0+0}{2}\right) = (9, 0)$
$(x_{mp\_AC}, y_{mp\_AC}) = \left(\frac{0+12}{2}, \frac{0+6}{2}\right) = (6, 3)$
$(x_{mp\_CB}, y_{mp\_CB}) = \left(\frac{12+18}{2}, \frac{6+0}{2}\right) = (15, 3)$

Next, we need to find the slope of all three lines. Recall the slope between two points, (x_1, y_1) and (x_2, y_2) is given as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$m_A = \frac{3-0}{15-0} = \frac{3}{15} = \frac{1}{5}$
$m_B = \frac{3-0}{6-18} = \frac{3}{-12} = -\frac{1}{4}$
$m_C = \frac{0-6}{9-12} = \frac{-6}{-3} = 2$

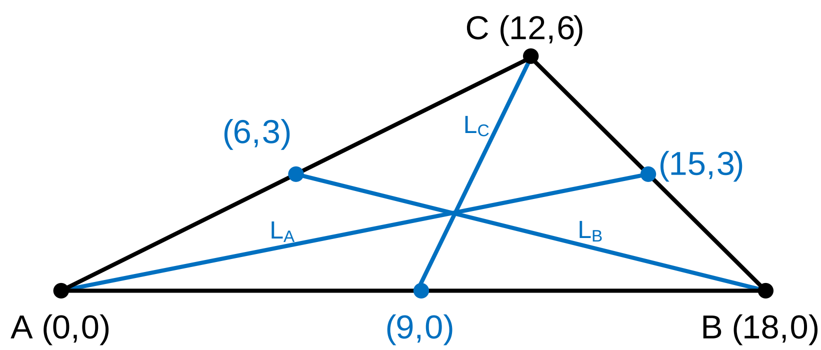
Finally, we can use the point slope formula to find the equation of each line. Recall, given a point on a line, (x_1, y_1), and the slope, m, the equation is given as:

$$y = m(x - x_1) + y_1$$

$(0,0), \quad m_A = \frac{1}{5}$	$y_A = \frac{1}{5}(x - 0) + 0$ $\boldsymbol{y_A = \frac{1}{5}x}$
$(18,0), \quad m_B = -\frac{1}{4}$	$y_B = -\frac{1}{4}(x - 18) + 0$ $\boldsymbol{y_B = -\frac{1}{4}x + \frac{9}{2}}$
$(12,6), \quad m_C = 2$	$y_C = 2(x - 12) + 6$ $\boldsymbol{y_C = 2x - 18}$

The three lines, named by their vertex, are shown in blue below.

Median Lines



Altitude Lines:

Recall the altitude lines go from one of the vertices to a point on the opposite side, such that it forms a 90° angle. In this case we can start by finding the slope of the lines. First recall that slopes of perpendicular lines are related as follows:

$$m_2 = -\frac{1}{m_1}$$

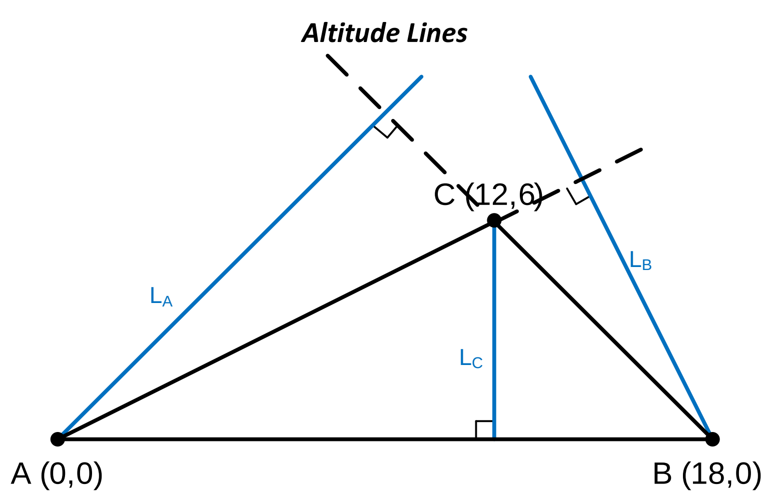
Therefore, we can find the slope of our lines by first finding the slope of each side of the triangle. From that we can find the slope of our altitude lines.

Slope of triangle segments	***Slope of corresponding altitude lines***
$m_{AB} = \frac{0-0}{18-0} = \frac{0}{18} = 0$	$m_{AB\_alt} = -\frac{1}{0} = undefined$
$m_{AC} = \frac{6-0}{12-0} = \frac{6}{12} = \frac{1}{2}$	$m_{AC\_alt} = -\frac{1}{1/2} = -2$
$m_{CB} = \frac{0-6}{18-12} = \frac{-6}{6} = -1$	$m_{CB\_alt} = -\frac{1}{-1} = 1$

Finally, we can again use the point slope formula to find the lines with the slopes from above and corresponding vertices.

$(12,6)$, $m_{AB\_alt} = undefined$	$\boldsymbol{x_C = 12}$
$(18,0)$, $m_{AC\_alt} = -2$	$y_B = -2(x-18)+0$ $\boldsymbol{y_B = -2x + 36}$
$(0,0)$, $m_{CB\_alt} = 1$	$y_A = 1(x-0)+0$ $\boldsymbol{y_A = x}$

The lines are again shown below in blue.



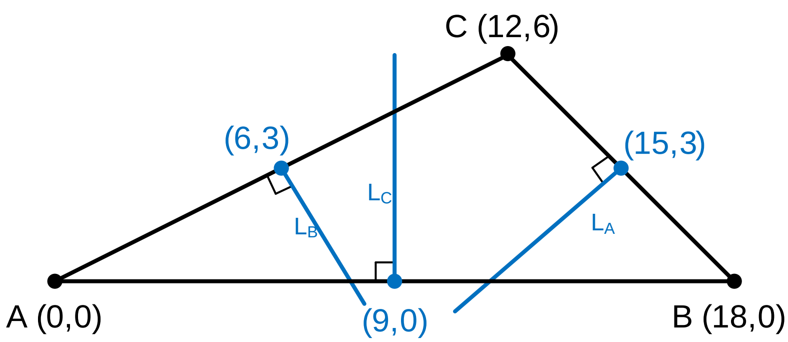
Perpendicular Bisector Lines:

Recall the perpendicular bisector lines starts from the midpoint of each segment of the triangle and move away at a 90° angle. Since these lines are perpendicular to the segments, they have the same slopes as the altitude lines we already found above. The only difference is that we need to use the midpoint as the point to create the lines.

$(9{,}0)$, $m_{AB\_alt} = undefined$	$\boldsymbol{x_C = 9}$
$(6{,}3)$, $m_{AC\_alt} = -2$	$y_B = -2(x-6)+3$ $\boldsymbol{y_B = -2x+15}$
$(15{,}3)$, $m_{CB\_alt} = 1$	$y_A = 1(x-15)+3$ $\boldsymbol{y_A = x-12}$

The lines are again shown below in blue.

Perpendicular Bisector Lines



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