Geometry Notes:

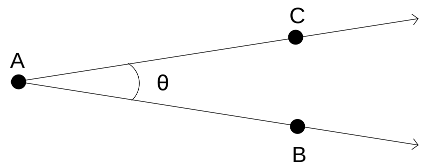
Part 1: Basic Definitions for Lines and Angles

Line: A line extends forever in both directions. $\overleftrightarrow{AB}$

Line Segment: A line segment is a part of a line. It has two endpoints. $\overline{AB}$

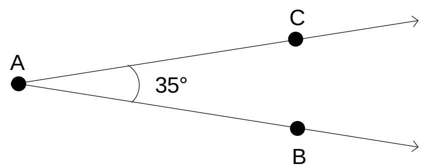
Ray: A ray starts at one point and continues in one direction forever. $\overrightarrow{AB}$

Angle: An angle is a figure formed by two rays, called the sides, that share a common endpoint, called the vertex. Two side rays: $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Measure of angle: θ

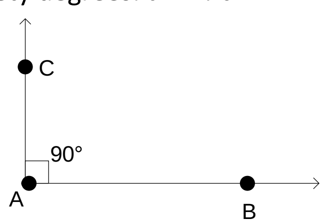


Types of Angles

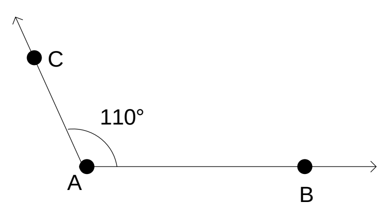
Acute: Angles in the range $0^{\circ} \leq \theta < 90^{\circ}$



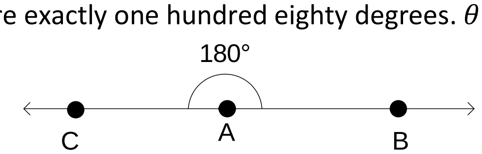
Right: An angle that measure exactly ninety degrees: $\theta = 90^{\circ}$



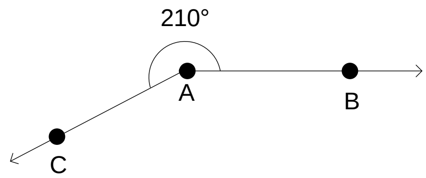
Obtuse: Angles in the range $90^{\circ} < \theta < 180^{\circ}$.



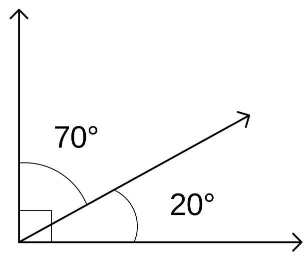
Straight: An angle that measure exactly one hundred eighty degrees. $\theta = 180^{\circ}$



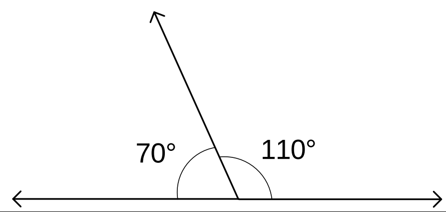
Reflex: Angles in the range $180^{\circ} < \theta < 360^{\circ}$.



Complementary Angle: When two angles add to 90° they are called complementary.



Supplementary Angle: When two angles add to 180° they are called complementary.



Part 2: Equality and Congruence

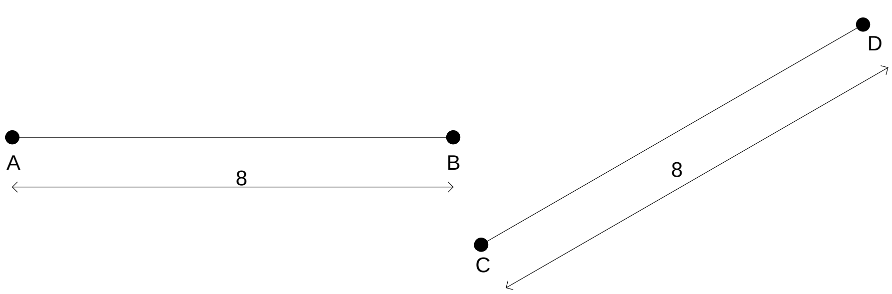
Congruence: Is a relationship between '*objects*': E.g., Line segments, lines, rays, angles, triangles.

Equality: Is a relationship between '*numbers*': E.g., lengths of line segments, measure of angles.

**Note:

A line segments is denoted as $\overline{AB}$, and the corresponding measure of the line segment is denoted as AB.

An angle is denoted as $\angle ABC$, and the corresponding measure of the angle is denoted as $m\angle ABC$.



Equality Relationship	***Congruent Relationship***
$AB = CD$	$\overline{AB} \cong \overline{CD}$

<u>Properties of Equality:</u> Mostly used to solve algebraic equations.

Addition Property of Equality	If $a = b$, then $a + \boldsymbol{c} = b + \boldsymbol{c}$
Subtraction Property of Equality	If $a = b$, then $a - \boldsymbol{c} = b - \boldsymbol{c}$
Multiplication Property of Equality	If $a = b$, then $a\boldsymbol{c} = b\boldsymbol{c}$
Division Property of Equality	If $a = b$, then $a/\boldsymbol{c} = b/\boldsymbol{c}$
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$ then $b = a$
Transitive Property of Equality	If $a = \boldsymbol{b}$ and $\boldsymbol{b} = c$ then $a = c$
Substitution Property of Equality	If $a = b$ then "a" can be substituted for "b" in any equation or expression.

<u>Properties of Congruence:</u> Mostly use to solve geometric proofs.

Reflexive Property of Congruence	Any geometric object is congruent to itself. e.g. $\overrightarrow{FG} = \overrightarrow{FG}$
Symmetric Property of Equality	If one geometric object if congruent to a second, then the second is congruent to the first. e.g. If $\overrightarrow{AB} = \overrightarrow{CD}$ then If $\overrightarrow{CD} = \overrightarrow{AB}$
Transitive Property of Equality	If one geometric object is congruent to a second, and the second is congruent to a third, then the first is congruent to the third. e.g., If $\overrightarrow{AB} = \boldsymbol{\overrightarrow{CD}}$ and $\boldsymbol{\overrightarrow{CD}} = \overrightarrow{EF}$, then $\overrightarrow{AB} = \overrightarrow{EF}$

Part 3: Useful Definitions, Properties, and Theorems/Postulates

Definition of Congruence	When two geometric objects have the exact same size and shape.
Definition of Midpoint	The point that divides a line segments into two congruent segments.
Definition of Segment Bisector	A line, ray, or segment that cuts another line segment into two congruent segments.
Definition of Perpendicular Segment Bisector	As segment bisector that results in two right angles.
Perpendicular Bisector Theorem	If a point lies on the perpendicular bisector of a line segment, then the point is equidistant from the endpoints of the segment.
Perpendicular Bisector Converse Theorem	If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.
Segment Addition Postulate	If point B is between A and C, the $AB + AC = AC$
Definition of Angle Bisector	A ray that divides an angle into two congruent angles.
Definition of Complementary Angles	Two angles whose measures sum to 90°.
Definition of Supplementary Angles	Two angles whose measures sum to 180°.
Definition of Adjacent Angles	2 angles with same vertex and common side but have no common interior points.
Angle Bisector Theorem	If a point lies on an angle bisector, then it is equidistant from the two sides of the angle. *** NOTE: the distance must be measured along the segments that are PERPENDICULAR to the sides on the angle*.
Angle Bisector Converse Theorem	If a point is equidistant from 2 sides of an angle, (*measured along the segments that are PERPENDICULAR to the sides on the angle*), then the point lies on the angle bisector of the angle.
Angle Addition Postulate	If point C is in the interior of $\angle AOD$, then $m\angle AOC + m\angle COD = m\angle AOD$
Definition of Linear Pair Angles	2 adjacent angles whose noncommon side form opposite rays.
Linear Pair Postulate	If 2 angles form a linear pair, then they are supplementary, i.e., that add to 180°
Definition of Vertical Angles	Two angles that are formed across from each other when two rays, lines, or line segments cross.
Vertical Angles Theorem	Vertical Angles are congruent.
Definition of Colinear	Points, segments, or rays, that lie on same line.
Definition of Coplanar	Points, segments, rays, or lines, that lie on the same plane.

Part 3 Continued: Properties of Parallel Lines and Proving Lines are Parallel

Definition of Perpendicular Lines	Lines that intersect each other to form right angles. Note: Right angles measure 90°.
Property of Perpendicular Lines Theorem	If 2 coplanar lines are perpendicular to the same line, then they are parallel to each other. $if\ a \perp b\ and\ c \perp b, \quad then\ a \parallel c$
Perpendicular Postulate	If there is a line and a point NOT on that line, then there is EXACTLY ONE line that goes through the given point AND is PERPENDICULAR to the original line.
Miscellaneous Perpendicular Theorems	• Perpendicular lines interstice to form FOUR right angles. • All right angles are congruent. • If two lines intersect to from a pair of congruent adjacent angles, then the lines are perpendicular.
Definition of Opposite Rays	2 rays on the same line with same initial point but point in opposite directions.
Definition of Intersecting Lines	2 coplanar lines that have exactly one point in common.
Definition of Oblique Lines	2 lines that intersect but do not form right angles.
Definition of Skew Lines	2 lines that do not lie in the same plane. (Note: 3D only – do not intersect and are not parallel)
Definition of Parallel Lines	2 coplanar lines that do not intersect.
Transitivity of Parallel Lines Theorem	If 2 lines are parallel to the same line, then they are parallel to each other. $if\ a \parallel b\ and\ b \parallel c, \quad then\ a \parallel c$
Parallel Postulate	If there is a line and a point NOT on that line, then there is EXACTLY ONE line that goes through the given point AND is PARALLEL to the original line.
Definition of a Transversal Line	A line that intersects ≥ 2 coplanar lines at DIFFERENT points.
Definition of Corresponding Angles	2 angles that occupy corresponding positions relative to lines, l, m, and the transversal.
Corresponding Angles Postulate	If two PARALLEL lines are cut by a transversal, then pairs of corresponding angles are congruent.
Corresponding Angles Converse Postulate	If two lines are cut by a transversal so that a pair of corresponding angles are congruent, then the lines are PARALLEL.

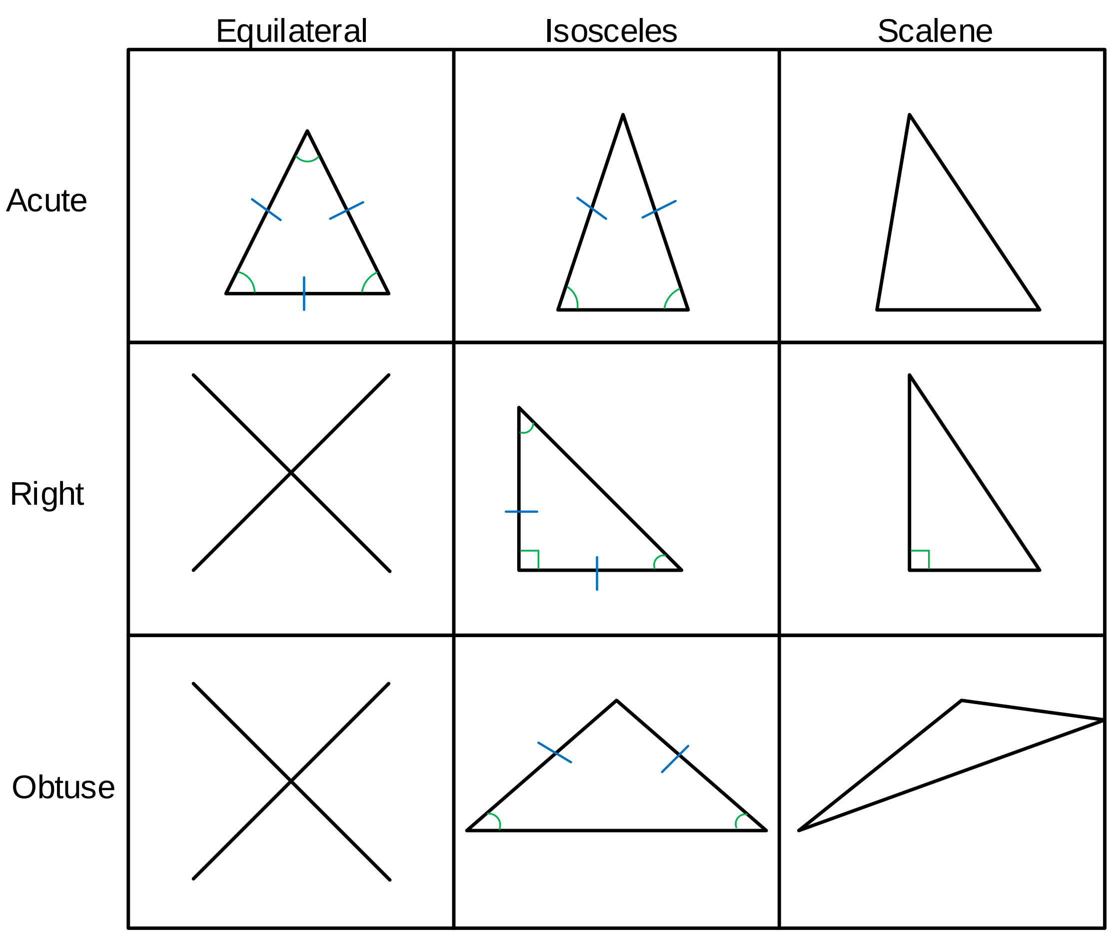
Part 3 Continued: Properties of Parallel Lines and Proving Lines are Parallel

Definition of Alternate Interior Angles	2 angles that lie BETWEEN lines l and m, but are on the OPPOSITE sides of the transversal.
Alternate Interior Angles Theorem	If two PARALLEL lines are cut by a transversal, then pairs of alternate interior angles are congruent.
Alternate Interior Angles Converse Theorem	If two lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the lines are PARALLEL.
Definition of Alternate Exterior Angles	2 angles that lie OUTSIDE lines l and m, but are on the OPPOSITE sides of the transversal.
Alternate Exterior Angles Theorem	If two PARALLEL lines are cut by a transversal, then pairs of alternate exterior angles are congruent.
Alternate Exterior Angles Converse Theorem	If two lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the lines are PARALLEL.
Definition of Consecutive Interior Angles	2 angles that lie BETWEEN lines l and m, and are on the SAME sides of the transversal.
Consecutive Interior Angles Theorem	If two PARALLEL lines are cut by a transversal, then pairs of consecutive interior angles are SUPPLEMENTARY.
Consecutive Interior Angles Converse Theorem	If two lines are cut by a transversal so that a pair of consecutive interior angles are SUPPLEMENTARY, then the lines are PARALLEL.

Part 4: Triangle Definitions, Properties, and Theorems

Triangles are classified according to the following two criteria:

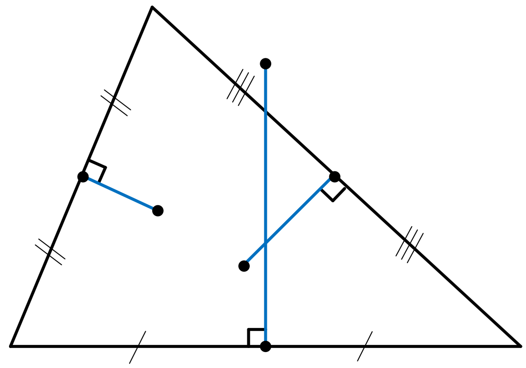
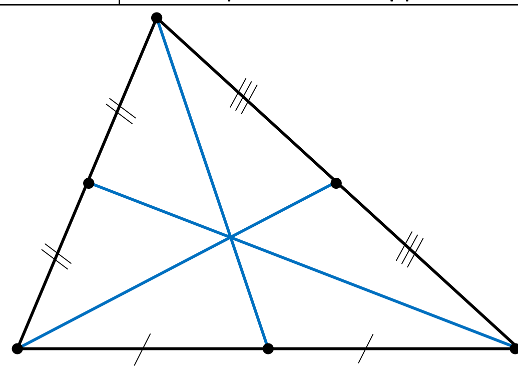
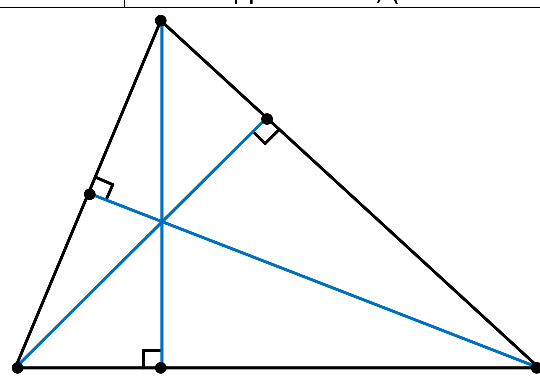
1. **Side Lengths**
   - ***Equilateral***
     - All three sides have equal length. This implies all angles are 60°
   - ***Isosceles***
     - Two sides have equal length. This implies two angles are the same.
   - ***Scalene***
     - No sides have same length. This implies all three angles are different.
2. **Interior Angles**
   - ***Acute***
     - All interior angles are acute.
   - ***Right***
     - One interior angle are Right.
   - ***Obtuse***
     - One interior angle are obtuse.

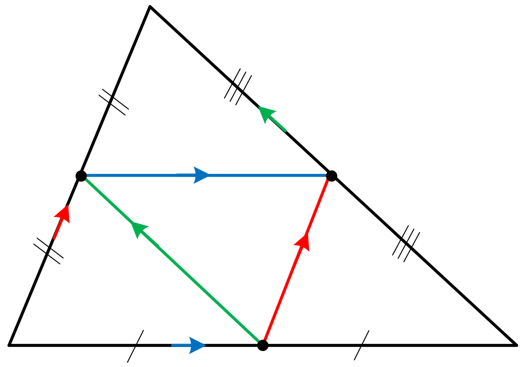


Part 4 Continued

Definitions for parts of a Triangle and Basic Theorems	
Opposite Side of a Triangle	The side that is ACROSS from an angle.
Adjacent Side of a Triangle	The side that is NEXT TO an angle.
Legs of a right triangle	The 2 sides that are ADJACENT to the right angle.
Hypotenuse of a right triangle	The side that is OPPOSITE to the right angle.
Legs of an isosceles triangle	The 2 congruent sides.
Base of an isosceles triangle	The non-congruent side.
Interior angle of a triangle	One of the 3 angles in a triangle.
Exterior angle of a triangle	An angle that forms a linear pair with one of the interior angles.
Angle Sum Theorem	The sum of the measure of the 3 interior angles of a triangle is 180°
Acute Angles Right Triangle Theorem	The acute angles of a right triangle are complementary.
Exterior Angles Theorem	The measure of an exterior angle of a triangle is equal to the sum of the 2 non-adjacent interior angles.
Base Angles Theorem	If 2 sides of a triangle are congruent, then the 2 angles across from them are also congruent.
Base Angles Converse Theorem	If 2 angles of a triangle are congruent, then the 2 sides across from them are also congruent.
Third Angle Theorem	If 2 angles of ONE triangle are congruent to 2 angles from a SECOND triangle, then the 3^{rd} angles are also congruent.

Ways to Prove two Triangles are Congruent	
SSS – Side-Side-Side Congruence Postulate	If 3 sides of ONE triangle are congruent to 3 sides of a SECOND triangle, then the 2 triangles are congruent.
SAS – Side-Angle-Side Congruence Postulate	If 2 sides and an included angle of ONE triangle are congruent to 2 sides and an included angle of a SECOND triangle, then the 2 triangles are congruent.
ASA – Angle-Side-Angle Congruence Postulate	If 2 angles and an included side of ONE triangle are congruent to 2 angles and an included side of a SECOND triangle, then the 2 triangles are congruent.
AAS – Angle-Angle-Side Congruence Postulate	If 2 angles and a non-included side of ONE triangle are congruent to 2 angles and the CORRESPONDING non-included side of a SECOND triangle, then the 2 triangles are congruent.
HL – Hypotenuse-Leg Congruence Theorem	If the hypotenuse and a leg of one RIGHT triangle are congruent to the hypotenuse and the leg of a second RIGHT triangle, then the 2 right triangles are congruent.
CPCTC - Corresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent.	

Additional Properties and Theorems of Triangles	
Perpendicular Bisector of a Triangle	A segment that is *perpendicular* to one of the sides of triangle at that side's *midpoint*.
	
Median of a Triangle	A segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side
	
Altitude of a Triangle	A segment from a vertex of the triangle that is perpendicular to the opposite side, (**or a line containing the opposite side*).
	

<table>
<tr><th colspan="2">Additional Properties and Theorems of Triangles Continued</th></tr>
<tr><td>Midsegment of a Triangle</td><td>A segment that connects the midpoints of 2 sides of the triangle.</td></tr>
<tr><td colspan="2">
Midsegment Theorem for Triangles: The midsegment of a triangle connected 2 sides of the triangle is parallel to the third side.</td></tr>
<tr><td>Triangle “Long Side” Theorem</td><td>If one SIDE of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.</td></tr>
<tr><td>Triangle “Large Angle” Theorem</td><td>If one ANGLE of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.</td></tr>
<tr><td>Triangle Inequality Theorem</td><td>The sum of the lengths of ANY two sides of a triangle is ALWAYS greater than the length of the third side.

** The shortest distance between two points is a straight line.</td></tr>
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