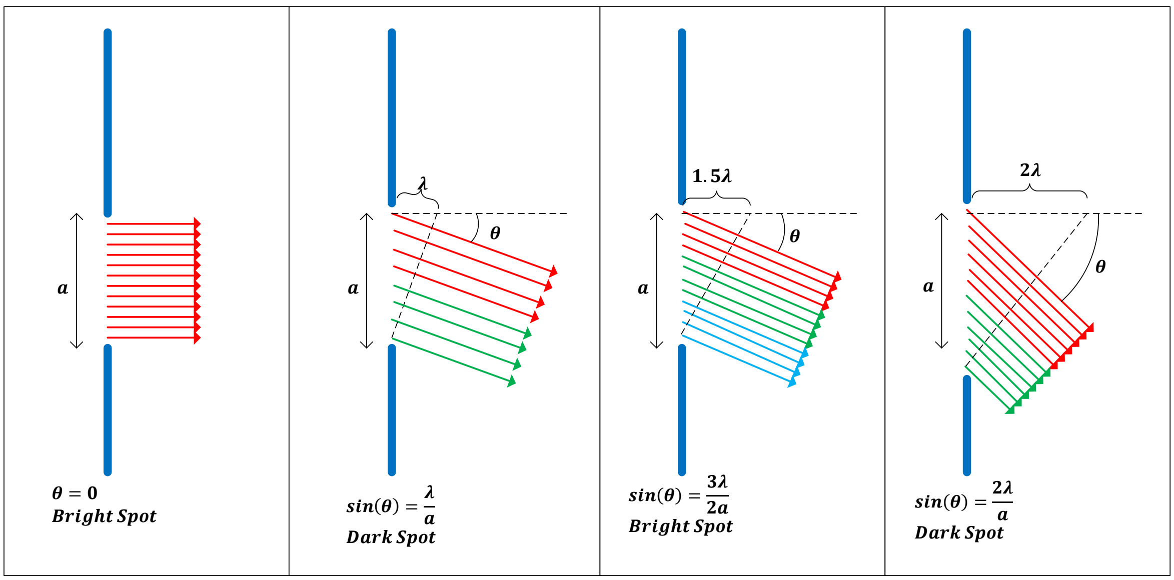
Wave Nature of Light – Diffraction and Polarization

In the previous lesson we saw that under certain circumstances light appears to behave as a wave. The wave nature of light was most clearly demonstrated with Young's double slit experiment, which showed how light from the two slits produced a wave-like interference pattern on a screen. To understand this phenomenon, we also briefly introduced the idea of diffraction, i.e. the bending of waves behind obstacle. In this lesson we will look at diffraction much more closely. We will also introduce another property of light called polarization.

Diffraction by a Single Slit

When analyzing Young's double slit experiment, we implicitly assumed that each slit was infinitesimally small. Let's see what happens if we use a single slit with a non-infinitesimal width. The figures below show light with wavelength λ passing through a single slit of width a.



The different pictures show light waves traveling through the slit at different angles. We analyze the four cases below to get a qualitative understanding of the light pattern that would appear on a screen placed in front of the slit.

- Case 1: $Path\ lengt\ differnce\ from\ top\ to\ bottom\ wave$: $\Delta x = 0$. **Bright**
  - Since all waves pass straight through the slit, they will arrive at the screen in phase and add constructively to produce a bright spot.
- Case 2: $Path\ lengt\ differnce\ from\ top\ to\ bottom\ wave$: $\Delta x = \lambda = a\,sin(\theta)$. **Dark**
  - For each wave in the bottom half of the slit there is a wave in the top half of the slit that will travel $\lambda/2$ further resulting in destructive interference and a dark spot on the screen.
  - To Illustrate we imagine eight evenly distributed waves paired such that each pair contains two waves with a path length different $\lambda/2$.
  - $[1\,\lambda/8\,, 5\lambda/8], [2\,\lambda/8\,, 6\,\lambda/8], [3\lambda/8\,, 7\,\lambda/8], [4\,\lambda/8\,, 8\lambda/8]$.

- Case 3: $Path\ lengt\ differnce\ from\ top\ to\ bottom\ wave$: $\Delta x = 1.5\lambda = a\,sin(\theta)$. **Bright**
  - For each wave in the bottom third of the slit there is a wave in the middle third of the slit that will travel $\lambda/2$ further resulting in destructive interference.
  - To Illustrate we can again imagine waves separated by $\lambda/8$. In this case there will be twelve such waves. We can again pair the first eight waves such that each pair contains two waves with a path length different $\lambda/2$.
  - $[1\,\lambda/8\,, 5\lambda/8], [2\,\lambda/8\,, 6\,\lambda/8], [3\lambda/8\,, 7\,\lambda/8], [4\,\lambda/8\,, 8\lambda/8]$.
  - The waves in the top third will *not* cancel and will arrive at the screen and produce a bright spot. However, not nearly as bright as the spot in case 1.
- Case 4: $Path\ lengt\ differnce\ from\ top\ to\ bottom\ wave$: $\Delta x = 2\lambda = a\,sin(\theta)$. **Dark**
  - For each wave in the bottom fourth of the slit there is a wave in the quarter just above it that will travel $\lambda/2$ further resulting in destructive interference. The same is true for the 3rd and 4th quarters. Therefore, all waves will destructively interfere, resulting in a dark spot on the screen.
  - This case can be illustrated much in the same way as the two previous cases.

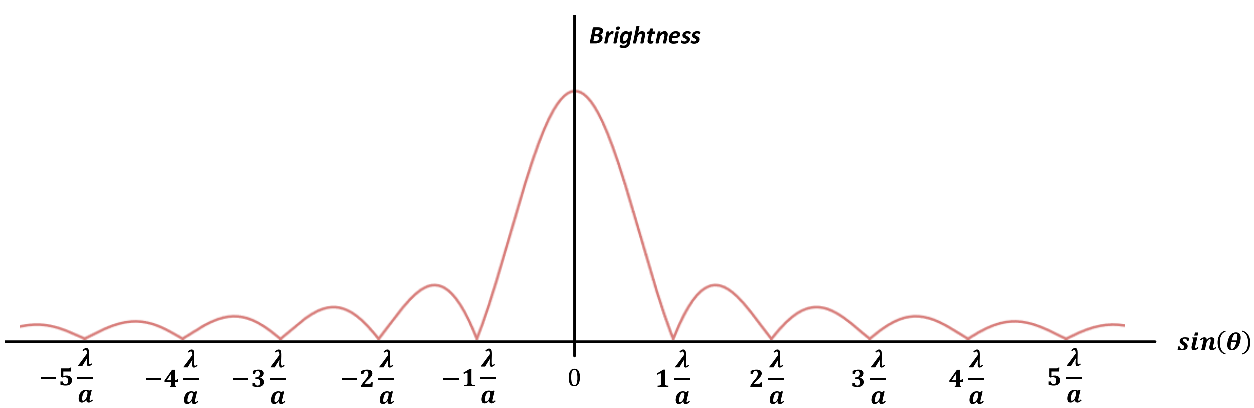
Continuing this pattern, we see that dark spots will occur when the path length difference is an integer multiple of the wavelength, except at zero.

Dark Spots
$\Delta x = m\lambda = a\,sin(\theta)\,, \qquad m = \pm 1, 2, 3 \ldots$

In addition to the bright spot at $\Delta x = 0$, bright spots occur when the path length difference is an integer multiple of half of the wavelength.

Bright Spots (Other than $\Delta x = 0$)
$\Delta x = (m + 1/2)\lambda = a\,sin(\theta)\,, \qquad m = \pm 1, 2, 3 \ldots$

However, it's important to note that the 'bright' spots become less bright as m gets larger. A sketch of the brightness with respect to $sin(\theta)$ is shown below as illustration.



Example 1: Light of wavelength 750 nm passes through a slit 0.001 mm wide. How wide if the central maximum in a.) degrees, b.) in centimeters on a screen that is 20 cm away?

Solution: As shown in the sketch above the central maximum is measured as twice the distance to the first minima. The angle measurement for the first minima can be solved as follows.

$$sin(\theta) = \frac{\lambda}{a}$$

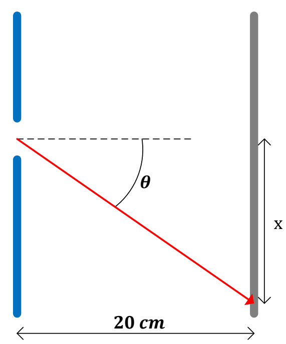
$$\theta = sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$= sin^{-1}\left(\frac{750}{1000}\right) = 48.6^{\circ}$$

The central maxima angular spread is then given as

$$\Delta\theta_{max1} = 2(48.6^{\circ}) = 97.2^{\circ}.$$

The distance along the screen to the first maxima can be found using the figure below.



$$x = 20\tan(48.6^{\circ}) = 22.7\ cm$$

The central maximum linear spread, Δx_{max1}, is then given as

$$\Delta x_{max1} = 2(22.7\ ) = 45.4\ cm$$

Example 2: a.) For a given wavelength, λ, how small must the slit width be for there to be no diffraction minima on the screen? b.) How small must the slit width be so that visible light exhibits no diffraction minima?

Solution: a.)

Referring again to the brightness sketch from above the minima, (on one side), occur as follows

$$sin(\theta) = \left[\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}, \frac{4\lambda}{a}, \ldots\right]$$

Since the maximum value of the sine function is 1.0 and since the terms listed above increase, we use the first minimum, i.e. $m = 1$, to determine the limiting value of a.

$$\frac{\lambda}{a} < 1 \rightarrow a > \lambda$$

Therefore, a must be larger than the wavelength for any diffraction minima to occur. In other words, if we want to avoid the diffraction minima, we must be sure that the slit width is smaller than the wavelength of light passing through the slit.

$$For\ a < \lambda, no\ diffraction\ minimam\ will\ occur$$

b.) Since the wavelength of visible light varies from 400 nm to 700 nm, no diffraction minima will occur with respect to visible light as long as the slit width is less than 400 nm.

Diffraction in Young's Double Slit Experiment:

When we first examined Young's double slit experiment, we intrinsically ignored the effects of diffraction by assuming the slits were of infinitesimal width. Although we didn't provide an expression for the intensity, (brightness), of the light on the screen in the previous section we state it here without proof.

$$I_{int}(\theta) = I_0\, cos^2\left(\frac{\pi d\theta}{\lambda}\right)$$

Where, we used the small angle approximation: $sin(\theta) = \theta$, I_0 is the intensity when $\theta = 0$, and d is the distance between the slits.

As you can tell from the equation, all of the bright spots have the same intensity. However, as we mentioned, this result assumes an infinitesimal slit width and therefore ignores the effects of diffraction. It is also possible to derive an expression for how the intensity of light varies because of diffraction. We again state the results below without proof.

$$I_{dif}(\theta) = I_0\left(\frac{sin\left(\frac{\pi a\theta}{\lambda}\right)}{\frac{\pi a\theta}{\lambda}}\right)^2$$

Where, we again used the small angle approximation: $sin(\theta) = \theta$, I_0 is the intensity when $\theta = 0$, and a is width of the slit.

To account for both the interference and the diffraction phenomena in Young's double slit experiment we multiple the two equations from above.

$$I(\theta) = I_0\left(\frac{sin\left(\frac{\pi a\theta}{\lambda}\right)}{\frac{\pi a\theta}{\lambda}}\right)^2 cos^2\left(\frac{\pi d\theta}{\lambda}\right)$$

The figures below illustrate the above concepts.

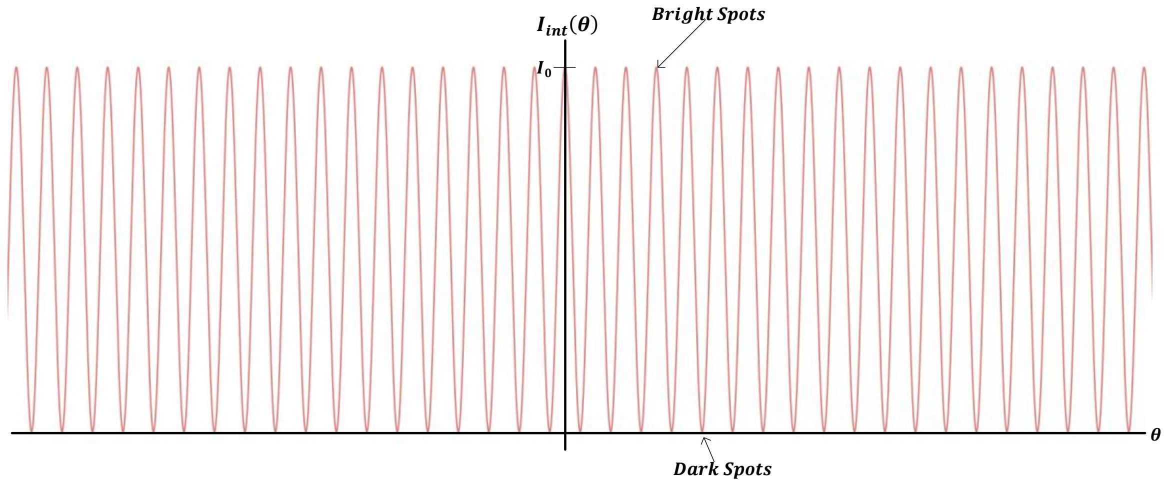


Figure 1. Interference Factor

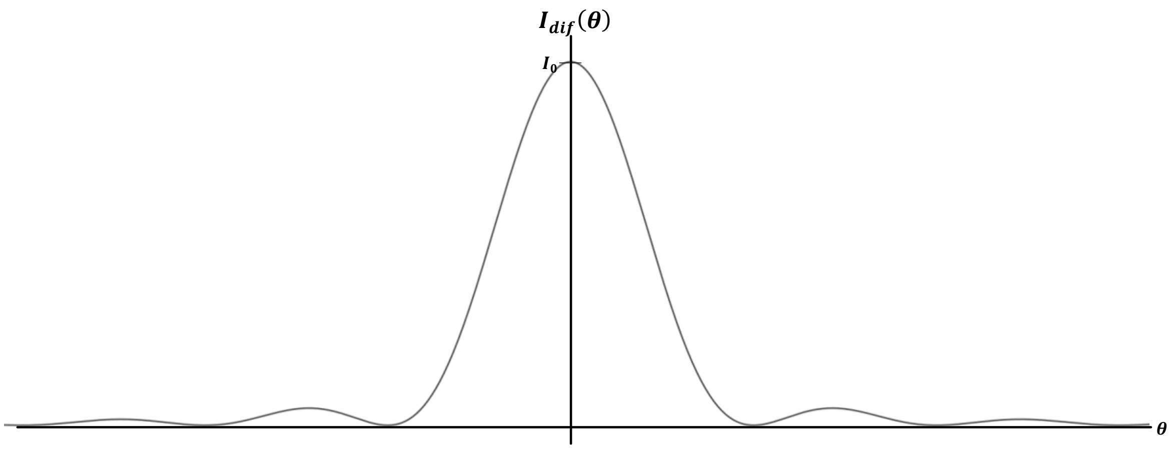


Figure 2. Diffraction Factor

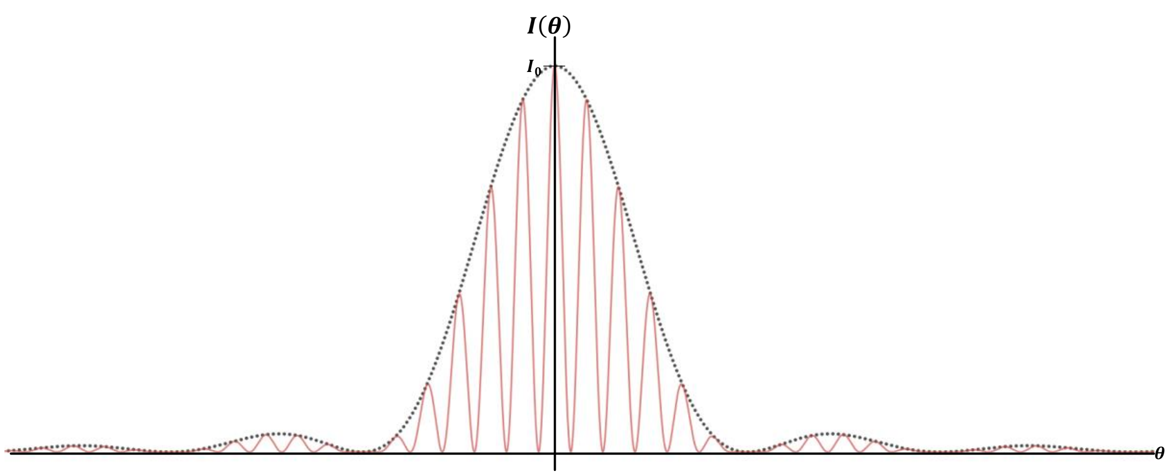


Figure 3. Interference and Diffraction Combined

The first figure shows the effects on the brightness on the screen due to interference alone, while the second figure shows the effects due to diffraction alone. The third figure shows the product of these two curves, which is the actual intensity that will be displayed on the screen for non-infinitesimal slit widths. The diffraction pattern acts as an envelope that limits the interference peaks. The example above illustrates the case when $d = 6a = 60\lambda$. Note as a gets smaller the main lobe in the diffraction factor graph gets larger. In the limit as $a \to 0$ the diffraction factor is a constant, i.e. $I_{dif}(\theta) = I_0$, and only the interference pattern remains.

Example 3: Design a double slit apparatus so that the central diffraction peak contains precisely fifteen bright spots.

Solution: From the previous section, using the small angle approximation, the angular values for the dark regions due to interference are given by

$$\theta_{int} = \frac{m\lambda}{d}$$

Similarly, using the small angle approximation for the first minima due to diffraction we have

$$\theta_{dif} = \frac{\lambda}{a}$$

In order to have fifteen interference peaks fit inside the main diffraction lobe we need seven on either side of the main interference peak, i.e. $m = 0$. Therefore we need the first minima from the diffraction pattern to occur at the eighth interference peak.

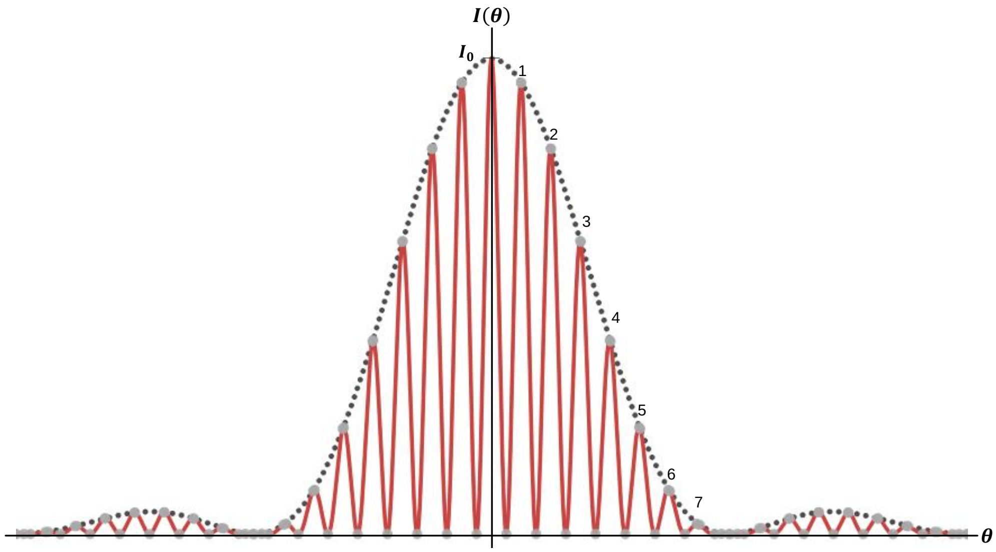
$$\theta_{dif,min1} = \theta_{int,8}$$
$$\frac{\lambda}{a} = \frac{8\lambda}{d}$$
$$d = 8a$$

Furthermore, recall that a must also be larger than the wavelength for diffraction to occur. For sake of argument let $a = 10\lambda$. Therefore, $d = 80\lambda$. Using these values in the intensity equation from above we have

$$I(\theta) = I_0 \left(\frac{sin\left(\frac{\pi(10\lambda)\theta}{\lambda}\right)}{\frac{\pi(10\lambda)\theta}{\lambda}} \right)^2 cos^2\left(\frac{\pi(80\lambda)\theta}{\lambda}\right)$$

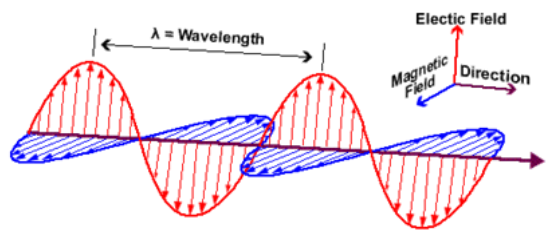
$$= I_0 \left(\frac{sin(10\pi\theta)}{10\pi\theta}\right)^2 cos^2(80\pi\theta)$$

The pattern is plotted below for illustration.

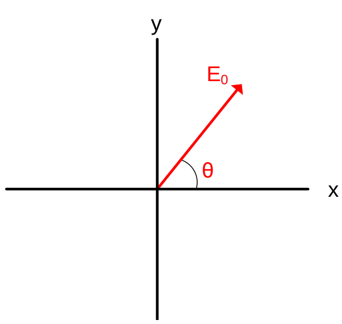


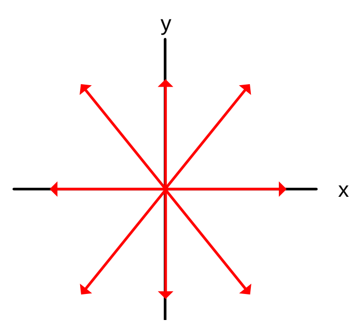
Polarization

Maxwell's theory predicts that light is a form of electromagnetic (EM) energy that travels through space as a transverse wave. The strength of the waves oscillate perpendicular to the direction of motion. More specifically, these waves consists of oscillating electric and magnetic fields that are themselves perpendicular to each other as illustrated below.

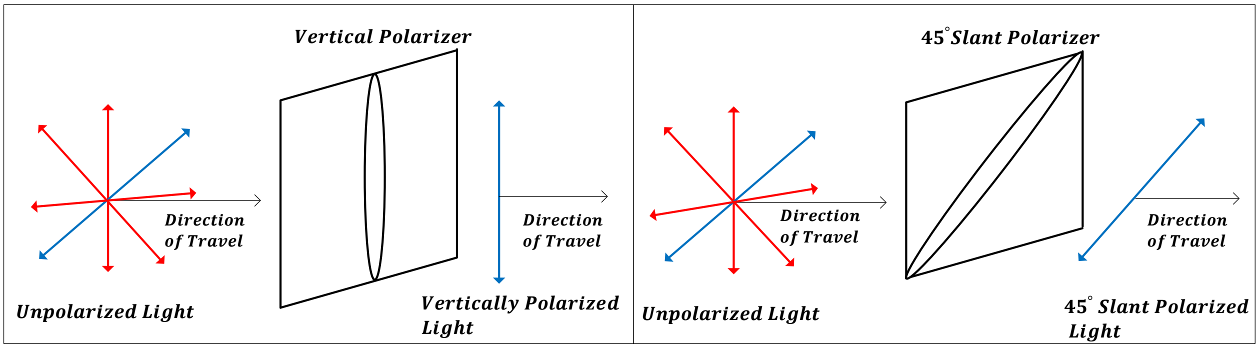


Polarization is a property of an EM wave that describes the orientation of the electric field, (since the magnetic field is always perpendicular to the electric field, polarization need only be referenced to one of the fields). In this lesson we consider linear polarization, where the electric field is oriented in a fixed direction. Since the direction of travel is always perpendicular to the orientation of the electric field, we can illustrate the polarization of an electromagnetic wave using the simple figure below on the left. The direction of travel is assumed in the positive z direction (out of the page), and the electric field is shown using standard vector notation.

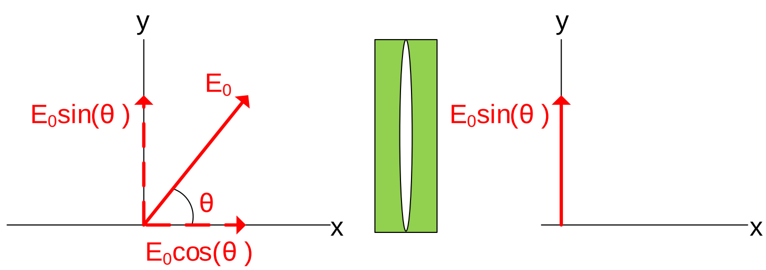




Light is not necessarily polarized when it is emitted from a source, instead it may have many different orientations simultaneously, as shown in the figure above on the right. An example is light from the sun or from an incandescent bulb. We refer to this as *unpolarized* light. Linearly polarized light can be obtained from unpolarized light using various materials, usually referred to as a polarizer. The simplest way to illustrate this is by imagining an object with a slit oriented in a certain position so that only one particular orientation of the waves can pass.



If a beam of θ° linearly polarized light strikes a vertical polarizer, the beam will emerge with a diminished magnitude, which can be determined using basic vector math.

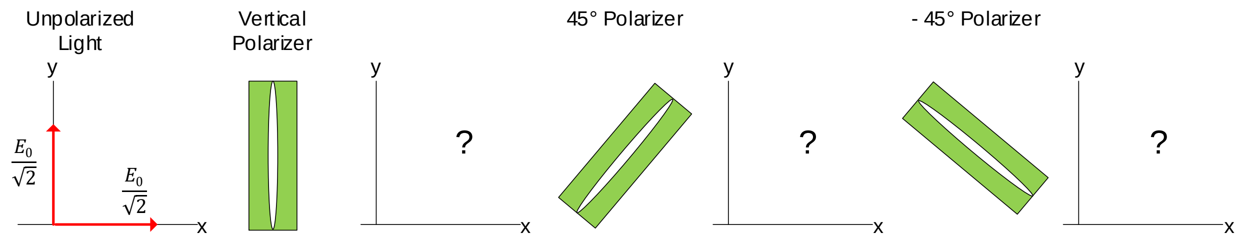


And since the intensity is proportional to the square of the magnitude, we can see the intensity of the vertically polarized beam is

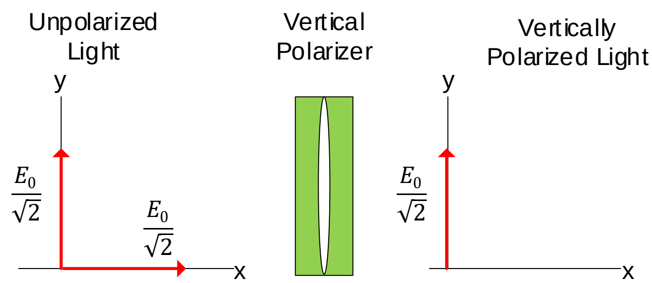
$$I = (E_0 \, sin(\theta))^2 = I_0 \, sin^2(\theta)$$

To describe unpolarized light more easily we can imagine that all of its random light directions are resolved into mutually orthogonal components, e.g. x and y components. On average unpolarized light can be thought of as two equal magnitude orthogonal beams. In this manner it is easy to see that when unpolarized light passes through a vertical or horizontal polarizer the intensity is reduced by a half. Moreover, if unpolarized light is passed through each of these polarizers in series, all light will be blocked. We illustrate these concepts with the examples below.

Example 4: Compute the intensity of the light at the outputs of each of the polarizers in the experiment shown below.



Solution: When light goes through the first polarizer only the vertical component survives.



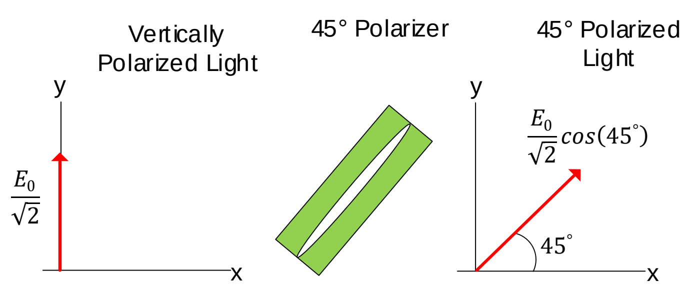
The intensity of the unpolarized light is

$$I_0 = \left(\frac{E_0}{\sqrt{2}}\right)^2 + \left(\frac{E_0}{\sqrt{2}}\right)^2 = {E_0}^2$$

The intensity of the light at the output of the first polarizer is then

$$I_1 = \left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{{E_0}^2}{2} = \frac{1}{2} I_0$$

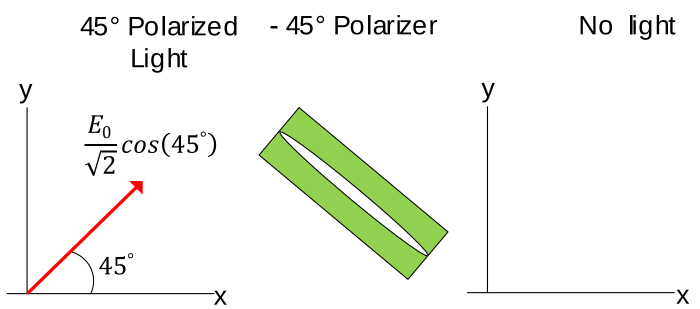
The figure below depicts what happens when this vertically polarized light goes through the second polarizer.



The intensity of the light at the output of the second polarizer is then

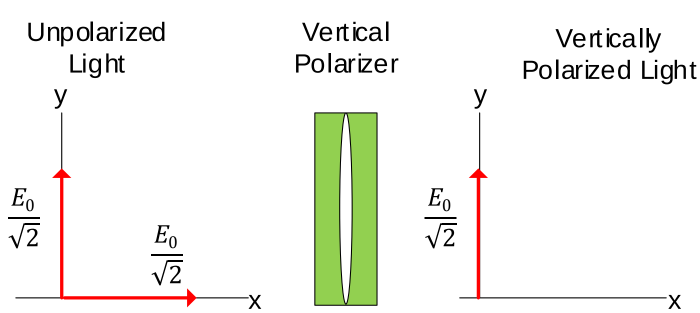
$$I_2 = \left(\frac{E_0}{\sqrt{2}} cos(45^{\circ})\right)^2 = \left(\frac{E_0}{\sqrt{2}} \frac{\sqrt{2}}{2}\right)^2 \frac{{E_0}^2}{4} = \frac{1}{2} I_1 = \frac{1}{4} I_0$$

Finally, when the 45° polarized light goes through a -45° polarizer no light passes. Note that the incident light and the polarizer are orthogonal.



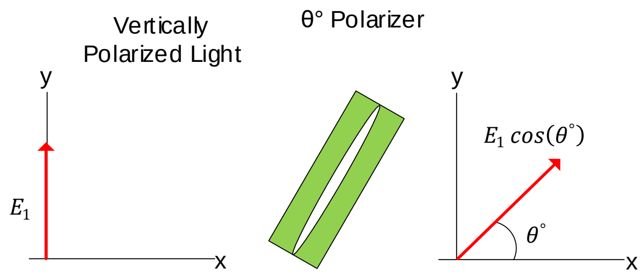
Example 5: At what angle should the axes of two polarizers be placed so as to reduce the intensity of unpolarized light to (a.) $1/3$, (b.) $1/10$?

Solution: In both cases we can start with a vertical polarizer, which we know from the previous example reduces the intensity by one half.



$$I_1 = \left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{{E_0}^2}{2} = \frac{1}{2}I_0$$

The next polarizer needs to reduce the intensity $2/3$ and $1/5$ or equivalently reduce the magnitude by $\sqrt{2/3}$ and $\sqrt{1/5}$ respectively.



The angle for the two cases are solved as shown below.

(a.) $\boldsymbol{I_2 = 1/3\, I_0}$	**(b.)** $\boldsymbol{I_2 = 1/10\, I_0}$
$E_2 = E_1\, cos(\theta)$ $\cancel{E_1}\sqrt{2/3} = \cancel{E_1} cos(\theta)$ $\theta = cos^{-1}\left(\sqrt{2/3}\right)$ $\theta \cong 35^{\circ}$	$E_2 = E_1\, cos(\theta)$ $\cancel{E_1}\sqrt{1/5} = \cancel{E_1} cos(\theta)$ $\theta = cos^{-1}\left(\sqrt{1/5}\right)$ $\theta \cong 63^{\circ}$

Final Summary for Wave Nature of Light – Diffraction and Polarization

Diffraction of Light
Diffraction is used to describe the fact that light bends around objects it passes and therefore spreads out after passing through a narrow slit. Light passing through a narrow slit of width a will produce a pattern of dark and bright spots that decrease in intensity, according to the following.

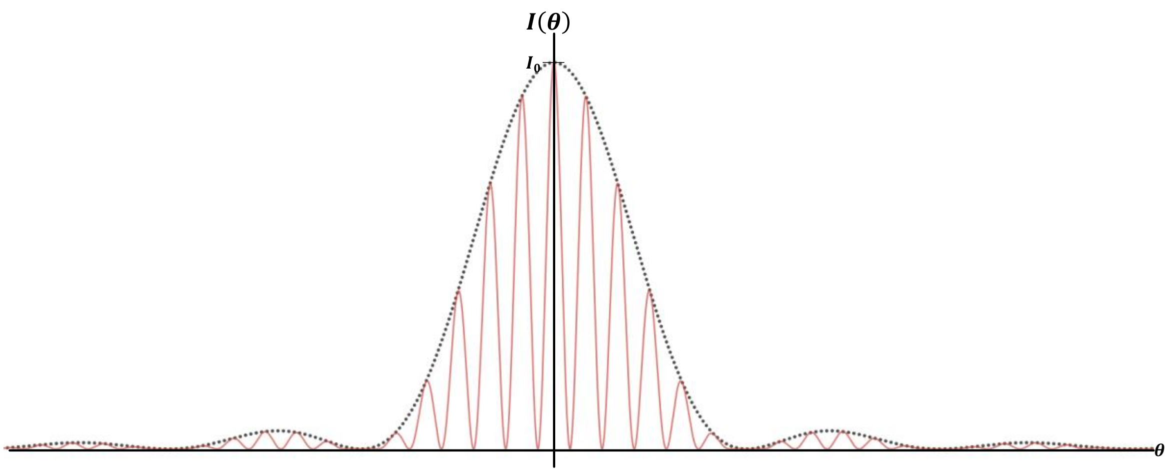
Dark Spots
$\Delta x = m\lambda = a\,sin(\theta)\,, \qquad m = \pm 1, 2, 3...$

In addition to the bright spot at $\Delta x = 0$, bright spots occur when the path length difference is an integer multiple of half of the wavelength.

Bright Spots (Other than $\Delta x = 0$)
$\Delta x = (m + 1/2)\lambda = a\,sin(\theta)\,, \qquad m = \pm 1, 2, 3...$

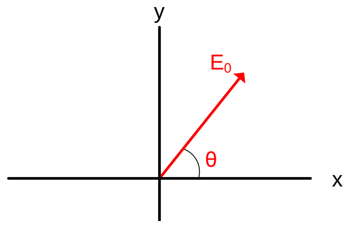
Young's Double-Slit Experiment
If we account for diffraction, (by using a non-infinitesimal slit width), the intensity of light on a screen is given as the product of the intensity due to interference and the intensity due to diffraction.

$$I(\theta) = I_0 \left(\frac{sin\left(\frac{\pi a \theta}{\lambda}\right)}{\frac{\pi a \theta}{\lambda}} \right)^2 cos^2\left(\frac{\pi d \theta}{\lambda}\right)$$

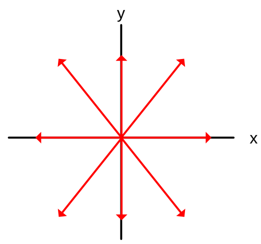


Polarization

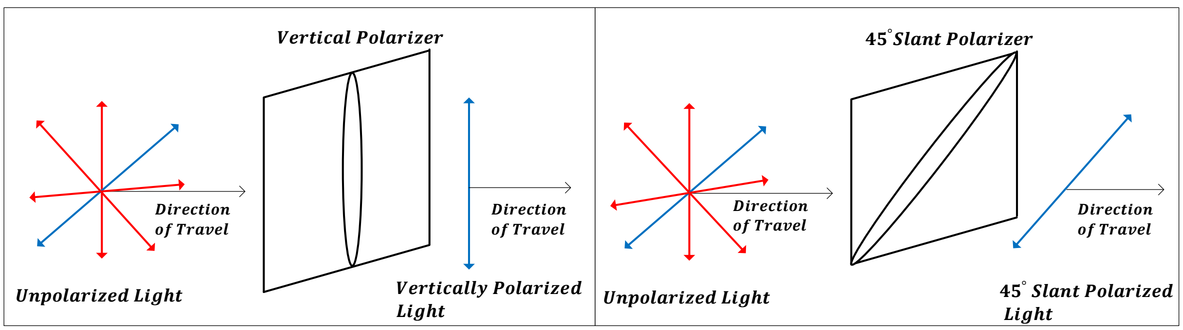
Polarization is a property of an EM wave that describes the orientation of the electric field. Linear polarization refers to the orientation in a fixed direction.



Unpolarized light, e.g. light from the sun or an incandescent bulb, has many different orientations simultaneously.



A polarizer is a material that can be sued to polarize light.



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