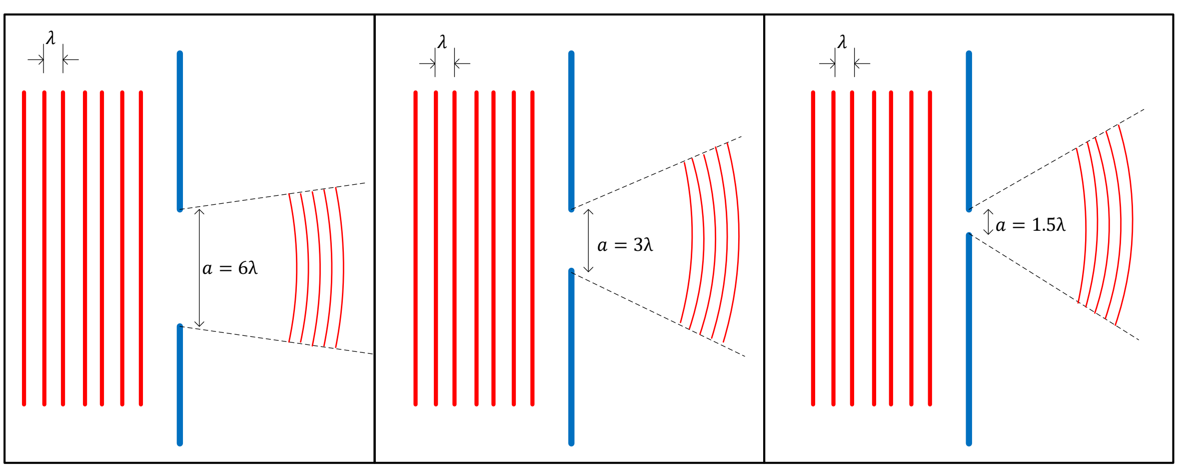
Wave Nature of Light - Interference

While studying mirrors and lenses in the previous two lessons we used the ray model of light. Modeling light as a ray considers light as a stream of particles and allowed us to understand the phenomena of reflection and refraction more easily. We also mentioned that, according to Maxwell's equations, light can also be modeled as a wave. Modeling light as a wave enables us to understand other light phenomena, such as interference and diffraction. In the next two lessons we will study these two phenomena as well as another aspect of light called polarization. In 1801 Thomas Young developed an experiment that very convincingly showed that light indeed behaves as if it were a wave. He showed that light exhibited the property of interference, which can most easily be associated with waves. To fully understand his experiment, we must also understand the idea of diffraction, which we discuss below.

Diffraction Introduction:

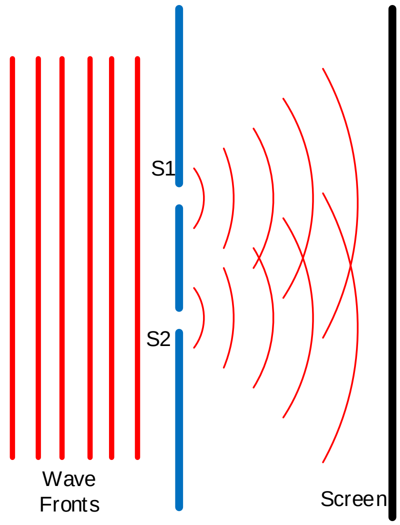
Diffraction is the bending of waves behind obstacles. We are all likely familiar with this phenomenon with regard to water waves. The picture below shows water waves passing through an opening. Notice how the wave flares out, diffracts, into the region beyond the barrier. This is exactly analogous to what Tomas Young observed in his experiments with light.

The figures below illustrate diffraction for a plane wave encountering a slit with a width that is on the order of the wavelength of the wave λ. The part of the wave that passes through the slit flares out beyond the edges of the opening. The figures also illustrate a key characteristic of diffraction: the narrower the slit, the more pronounced the diffraction. This particular phenomenon of light is what limits our ability to model light as a ray. However, as we can see from the figures if the slit is substantially larger than the wavelength of the incident light the diffraction is minimized and can be ignored.

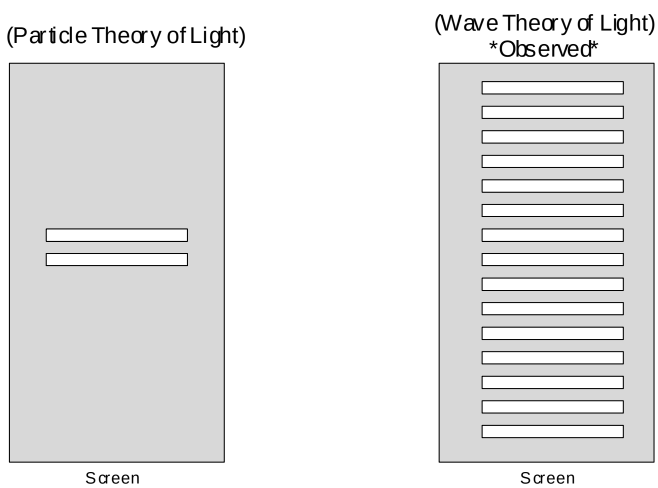


Young's Double-Slit Interference Experiment:

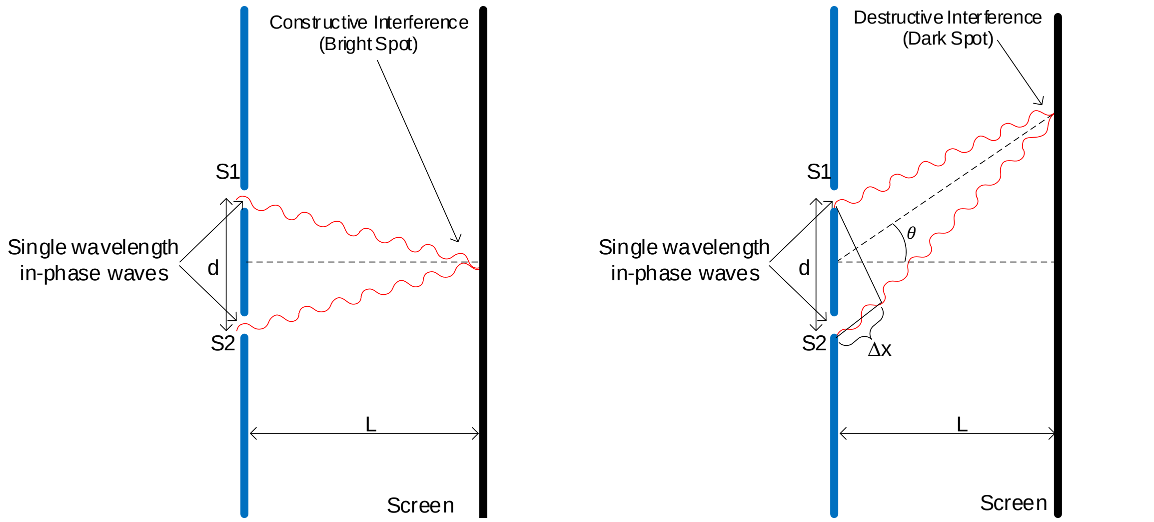
There has been a long history of debate about the nature of light. The ray model we used in the previous sections essentially treats light as a stream of particles that travel away from the source. Maxwell's equations treat light as if it traveled in the form of waves that spread outward from the source. This 'particle-wave' debate took place over many years with different experiments seeming to point towards one or the other theory. One of the most convincing experiments for the wave nature of light was Young's double slit experiment performed in 1801. The figure below illustrates the experiment performed.



Light from a single source falls on two closely spaced slits, $S1$ and $S2$. A screen is placed some distance away and the pattern of light is observed. If light consisted of a stream of particles, we would expect to see two bright lines on the screen, as shown in the figure below on the left. However, what Young observed instead was a series of bright lines as shown in the figure on the right. Young was able to explain this pattern as a wave-interference phenomenon.

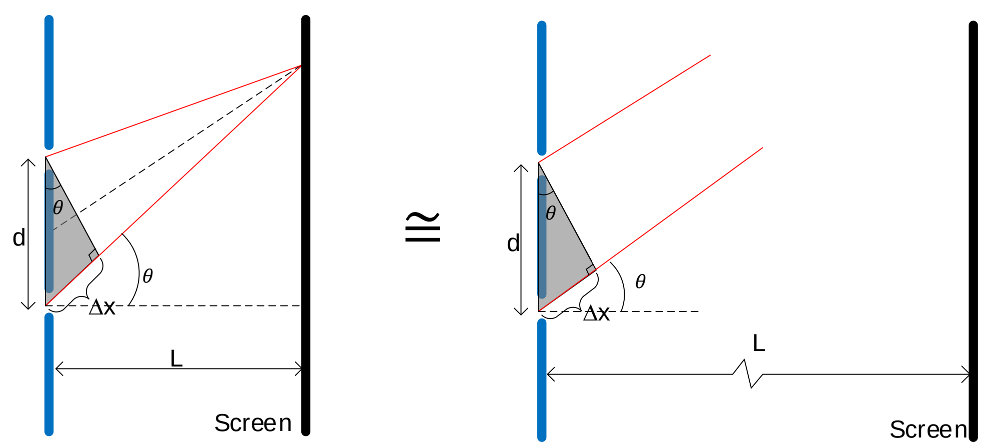


To see how the interference pattern is produced we use the two figures below, which show the light traveling as a wave through each slit to the different parts of the screen.



Waves with wavelength λ enter the two slits which are a distance d apart. The waves spread out in all directions but are shown for two specific angles. The figure on the left illustrates the case when $\theta = 0$. In this case each wave will travel the same distance. Therefore, when they arrive at the screen their crests and troughs are aligned, i.e. in-phase, and the waves will add to produce a larger amplitude wave, i.e. a bright spot. We refer to this as *constructive interference*. The figure on the right illustrates the case when $\theta \neq 0$, resulting in the two waves traveling a different distance. Therefore, when they arrive at the screen their crests and troughs may not be aligned. When the difference in distance is equal to half of a wavelength, $\Delta d = \lambda/2$, the crest of one wave aligns with the trough of the other and the waves add to zero, i.e. a dark spot. We refer to this as *destructive interference*. Since the waves are periodic there will be a series of bright and dark spots on the screen as the angle, and hence the path difference Δx, varies. Bright spots will occur whenever the path difference is an integer multiple of the wavelength, i.e. $\Delta x = m\lambda$ and dark spots will occur whenever the path difference is an integer multiple of half of a wavelength, i.e. $\Delta x = (m + 1/2)\lambda$.

In order to determine exactly where the bright and dark spots will appear, we should note first that the scale of the figures above are much exaggerated. In reality the distance, d, is very small compared to the distance to the screen, L. Therefore, the paths for the two waves can be approximated as parallel for all angles, θ. This approximation allows us to use the figure below to determine the path difference, Δx.



With the waves being parallel and θ being the angle they each make with the horizontal we can draw a right triangle as shown. The base of the triangle represents the difference in the path length, Δx, between the two waves, which can be written as a function of d and θ as follows.

$$\Delta x = d\,sin(\theta)$$

Using what we argued above with regard to the distances that cause constructive and destructive interference we can write the following key relationships.

Young's Double-Slit Wave Interference Relationships	
Constructive Interference (Bright Spots)	$d\,sin(\theta) = m\lambda, \quad m\ is\ any\ integer$
Destructive Interference (Dark Spots)	$d\,sin(\theta) = (m + 1/2)\lambda, \quad m\ is\ any\ integer$

Example 1: A replication of Young's experiment is set up with $d = 0.10\ mm$ and $L = 1.2m$. Light of wavelength $400\ nm$ is used. Find the locations of the bright spots on the screen.

Solution: The bright spots appear on the screen when the following condition holds.

$$d\,sin(\theta) = m\lambda$$
$$sin(\theta) = \frac{m\lambda}{d}$$

The first thing to notice is that since $sin(\theta) \leq 1$, there will be a limit to the number of bright spots on the screen. The maximum number can be found as follows:

$$\frac{m\lambda}{d} \leq 1 \rightarrow m \leq \frac{d}{\lambda}$$

In our case we find

$$m \leq \frac{0.10 \cdot 1E6}{400} = 250$$

The next question we can ask ourselves is "Are the spacings between the bright spots uniform?". Indeed, the spacings are *not* uniform since the sine function is non-linear. The angular values vary as the inverse sine function.

$$\theta_m = sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Which means the spacings get larger as m increases. However, we also know for small angles $sin(\theta) \cong \theta$. Using this approximation results in a linear relationship so that the spacings are indeed very close to uniform near the center of the screen.

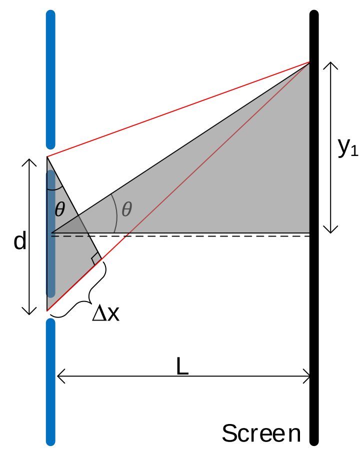
$$sin(\theta_m) \cong \theta_m = \frac{m\lambda}{d}$$

Focusing on the center of the screen the angular spacing is then computed using θ_1.

$$\theta_1 = \frac{\lambda}{d} = \frac{400}{0.10 \cdot 1E6}\left(\frac{180^{\circ}}{\pi}\right) \cong 0.23^{\circ}$$

Where we converted to degrees for convenience.

Next, to find the locations on the screen we can use a second right triangle as shown below.



Therefore,

$$y_m = L\,tan(\theta_m)$$

However, the same small angle approximation also applies to the tangent function. Therefore, we can write

$$y_m = L\theta_m = m\left(\frac{L\lambda}{d}\right) = m\left(\frac{1.2 \cdot 400\ x\ 10^{-9}}{0.1\ x\ 10^{-3}}\right) = m0.0048$$

The spacings between bright spots in the center of the screen are then $4.8\ mm$ apart.

In the next lesson we will see that the intensity of the bright spots decrease fairly rapidly and therefore the small angle approximations can be used in most practical applications.

Young's Double-Slit Wave Interference Small Angle Approximation Relationships		
Constructive Interference (Bright Spots)	$\theta_m = m\frac{\lambda}{d}$,	$y_m = m\frac{L\lambda}{d}$
Destructive Interference (Dark Spots)	$\theta_m = (m + 1/2)\frac{\lambda}{d}$,	$y_m = (m + 1/2)\frac{L\lambda}{d}$

Example 2: Describe the first order pattern that appears on the screen when white light is used in a double slit experiment where the two slits are $0.5\ mm$ apart and the screen is $2.5\ m$ away.

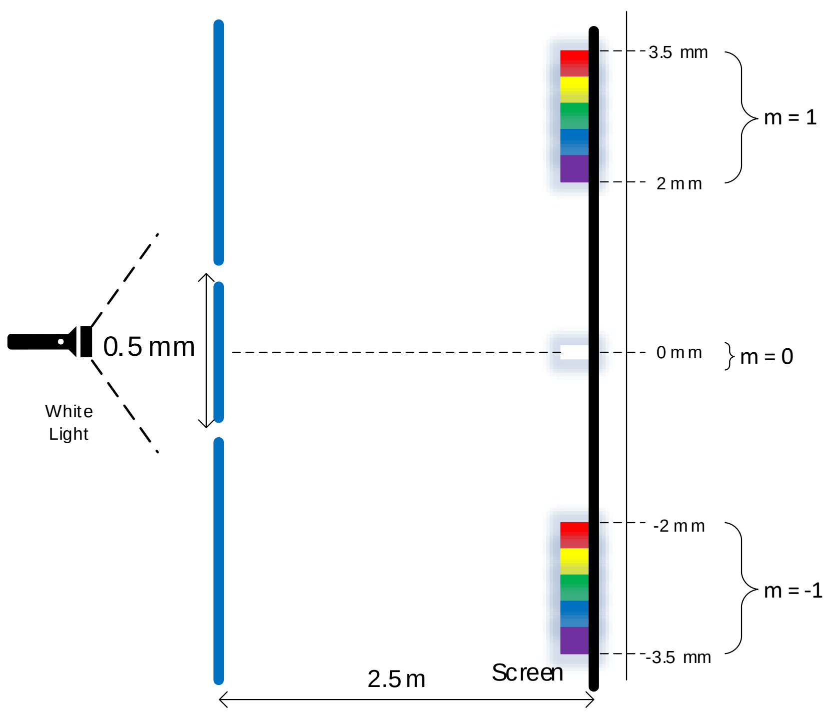
Solution: White light consists of wavelengths from about $400\ nm$, violet light, up to $700\ nm$, red light. The approximate constructive interference equation from example 1 is given as

$$y_m = m\frac{L\lambda}{d}$$

As you can see, the location of bright spots on the screen vary linearly with wavelength for any particular m. Therefore, the bright spots will spread into the various colors, i.e. wavelengths, across the screen in the familiar 'rainbow' pattern. Note however that this will not occur for the center bright spot, i.e. $m = 0$. The first rainbow pattern will start with the smallest wavelength, $400\ nm$, and end with the largest, $700\ nm$, with $m = 1$.

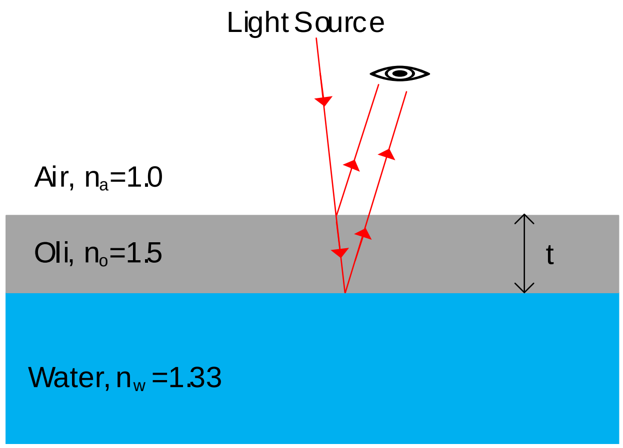
Start of first band	*End of first band*
$y_{1,400} = (1)\frac{L\lambda_{400}}{d}$ $= \frac{2.5 \cdot 400}{500000}$ $= 2\ mm$	$y_{1,700} = (1)\frac{L\lambda_{700}}{d}$ $= \frac{2.5 \cdot 700}{500000}$ $= 3.5\ mm$

Higher order bands are computed in a similar fashion. The figure below illustrates the first band pattern on the screen.



Interference in Thin Films:

Have you ever wondered what is causing the bright color patterns seen when you observe a thin film of oil resting on a puddle of water? How about the colors that sometimes appear to emanate from sunlit soap bubble? These phenomena are caused by interference. To illustrate let's take a closer look at the example below.



We'll start by assuming a single wavelength light source impinging on a thin film of oil with uniform thickness, t. When light strikes the top surface of the oil some is reflected, while the remaining refracts through the oil and subsequently reflects from the top surface of the water. When these two light waves reach the observer, one has traveled an extra distance of approximately $2t$. Similar to the double slit experiment these waves will add constructively when $2t = m\lambda$, and destructively when $2t = \left(m + \frac{1}{2}\right)\lambda$. For white light different colors, i.e. wavelengths, will experience varying degrees of constructive and destructive interference. Also note that the extra distance of $2t$ assumes the angle of incidence is small, i.e. the viewing angle is head on. However, as the viewing angle changes the extra distance becomes greater than $2t$, thereby changing the interference pattern. Finally, variations of thickness of the thin film will also affect interference pattern.

If we were to perform this experiment with known variables in an attempt to predict the interference behavior, we would notice that our predictions are sometimes not quite right. How can this be explained? One of the reasons may arise because of a phase change phenomenon related to reflected waves. The phenomenon can be predicted using Maxwell's equations; however, we state the results here without proof.

Reflecting surface n_1 to n_2	*Phase of Reflected Wave*
$n_2 > n_1$	Phase change of $\lambda/2$
$n_2 < n_1$	Phase change of 0

Using the example from above we see that the part of the wave that reflects from the surface of the oil is phase shifted by $\lambda/2$ with respect to the part that if refracted since $n_o > n_a$. The refracted wave then reflects from the surface of the water but since $n_w < n_o$ the wave is not phase shifted upon reflection. Since refracted waves are never phase shifted the two parts of the wave that arrive back at the observer begin their journey at the surface of the oil being out of phase by exactly $\lambda/2$. This, of course, would completely reverse our prediction of constructive and destructive waves.

Another reason for the potential prediction errors can be due to the fact that the wavelength of light is different in different mediums. To see this, we start by recalling that the speed of a wave in a medium with an index of refraction, n, is

$$v_n = \frac{c}{n}$$

We also know that the wavelength of any wave is related to the speed of a wave and its frequency, f, as follows

$$\lambda_n = \frac{v_n}{f}$$

Finally, we know that the speed of light, c, is constant in a vacuum, and is given as

$$c = f\lambda$$

Where, λ, is the wavelength in a vacuum.

Starting with the second equation and substituting for v_n and then c, we have

$$\lambda_n = \frac{v_n}{f} = \frac{c}{nf} = \frac{f\lambda}{nf} = \frac{\lambda}{n}$$

Therefore, if light travels from a medium where $n_1 = 1.0$, i.e. a vacuum, to a medium with an index of refraction, n_2, the wavelength of light in medium 2 is

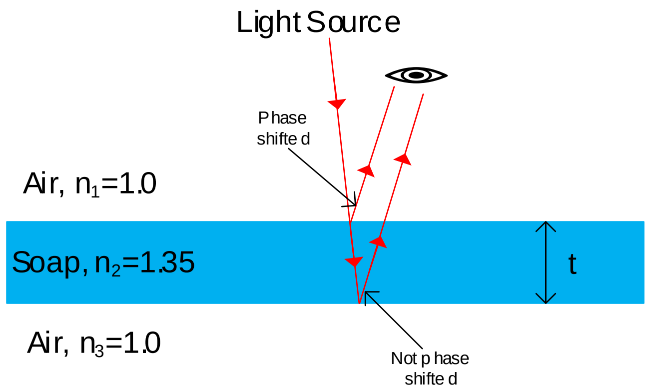
$$\lambda_{n_2} = \frac{\lambda}{n_2}$$

More generally we can write the following

$$\frac{\lambda_{n_2}}{\lambda_{n_1}} = \frac{n_1}{n_2} \rightarrow \lambda_{n_2} = \left(\frac{n_1}{n_2}\right)\lambda_{n_1}$$

Example 3: A soap bubble appears green, ($\lambda = 540\ nm$), at the point on its front surface nearest the viewer. What is the minimum thickness? Assume $n = 1.35$.

Solution: We start by drawing the figure below.



Since $n_2 > n_1$, the reflected wave undergoes a $\lambda/2$ phase shift at the top surface of the bubble, but not at the bottom surface. Therefore, green light will constructively interfere when the extra path length traveled inside the thin film, $2t$, is equal to an integer multiple of half of the wavelength inside the medium, $\left(m + \frac{1}{2}\right)\lambda_{n_2}$.

$$2t = \left(m + \frac{1}{2}\right)\lambda_{n_2}$$

Recall that $\lambda_{n_2} = \lambda/n_2$, and therefore the thickness of the soap bubble for constructive interference is given as

$$t = \frac{\left(m + \frac{1}{2}\right)\lambda}{2n_2}$$

With the minimum thickness occurring when $m = 0$.

$$t_{min} = \frac{\lambda}{4n_2} = \frac{540}{4 \cdot 1.35} = 100\ nm$$

Let's do a few more examples before summarizing what we learned in this lesson.

Example 4: Light of wavelength $680\ nm$ falls on two slits and produces an interference pattern in which the fourth-order bright spot is $38\ mm$ from the center spot on a screen $2.0\ m$ away. What is the separation of the two slits?

Solution: Although we can use the small angle approximation equations let's begin by using the initial relationships developed. In this case, the fourth-order bright spot is produced on the screen according the following.

$$d\,sin(\theta) = 4\lambda$$

Furthermore, the relationship between the location of the bright spot on the screen, y_4, and the angle, θ, is

$$y_4 = L\,tan(\theta)$$

Solving for θ and substituting into the first equation we can solve for the distance between the slits, d, as follows

$$d = \frac{4\lambda}{sin\left(tan^{-1}\left(\frac{y_4}{L}\right)\right)}$$

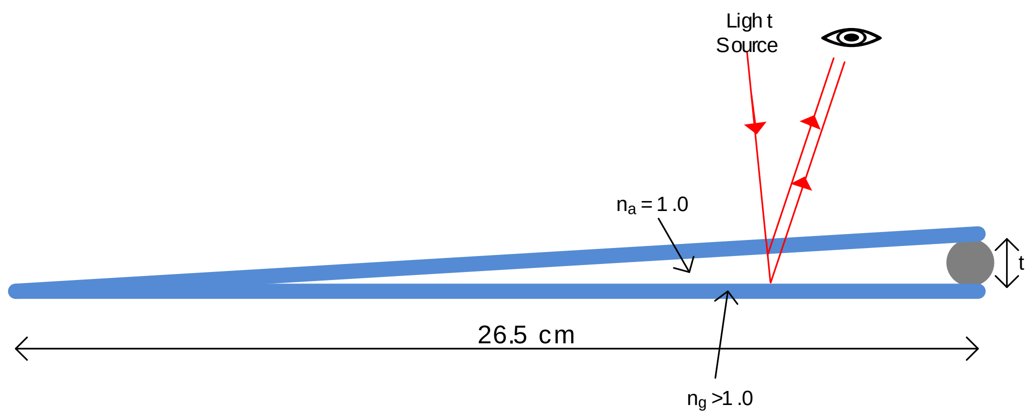
$$= \frac{4 \cdot 680}{sin\left(tan^{-1}\left(\frac{38\,x\,10^6}{2\,x 10^9}\right)\right)} \cong 143.184\,\mu m$$

Next, we compare this to the value using the small angle approximation equation solved for d.

$$d = 4\frac{L\lambda}{y_m} = 4\frac{2\,x 10^9 \cdot 680}{38\,x\,10^6} = 143.158\,\mu m$$

As you can see the small angle approximation gives us an answer that is very close to the value we computed using the more complex initial equation.

Example 5: A very fine wire $7.35\,x\,10^{-3}$ mm in diameter is placed between two flat glass plates as shown below. Light whose wavelength in air is 600 nm falls, (and is viewed), perpendicular to the plates, and a series of bright and dark bands is seen. Describe the number and order of the bright and dark bands that results. How far apart are the dark bands if the glass plates are 26.5 cm long?



Solution: Note in this case the phase shift occurs at the lower surface. Therefore, constructive interference (bright bands) and destructive interference (dark bands) occur according to the following

Bright Bands	**Dark Bands**
$2t = \left(m + \frac{1}{2}\right)\lambda$	$2t = m\lambda$

We can start by finding extra distance, (in terms of the number of wavelengths), that is traversed for the thickest part of the thin film.

$$\frac{2t}{\lambda} = \frac{2 \cdot 7.35\ x\ 10^3}{600} = 24.5$$

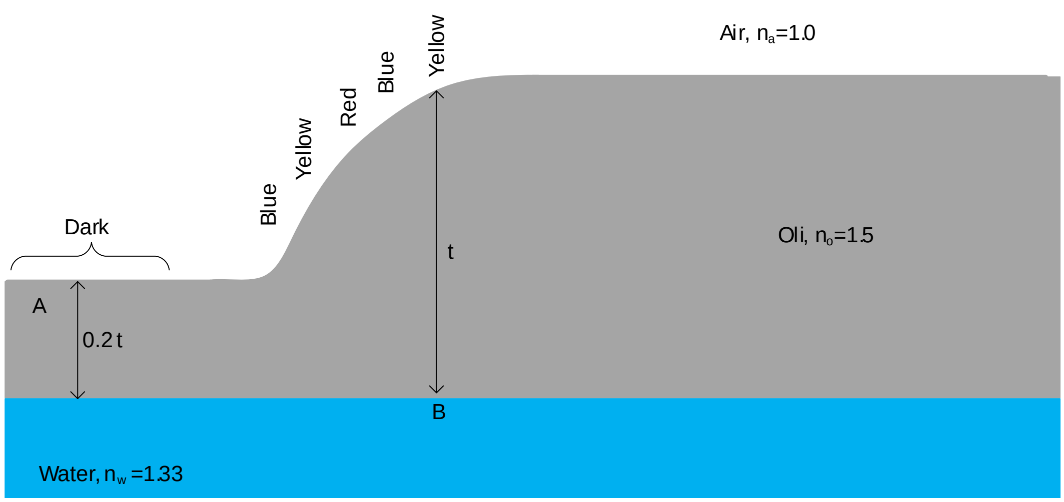
Indicating that at the thickest part the extra distance will result in a bright band with $m = 24$. Therefore, as we move along the glass until the thickness of the thin film goes to zero there will be 25 dark bands and 25 bright bands for $m = \ 24, 23, \ldots\ 0$.

Assuming the thin film varies linearly from t to 0 over $26.5\ cm$ the bands will be evenly distributed. Therefore, the distance between the dark (or bright) bands is

$$\Delta x\ = \frac{26.5}{24} \cong 1.1\ cm$$

Example 6: A thin film of oil ($n_o\ = 1.5$) with varying thickness floats on water ($n_w\ = 1.33$). When it is illuminated from above with white light, the reflected colors are as shown in the figure below. In air the wavelength of yellow light is $580\ nm$.

a. What is the oil's thickness, t, at point B?
b. The oil's thickness at point A is 20% of that at point B. Prove that there are no reflected visible colors seen at point A.



Solution: Since $n_o > n_a$ and $n_w < n_o$ the reflected path will undergo one $\lambda/2$ phase shift, resulting in wavelength dependent constructive interference as follows:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_o}$$

The first time we see the yellow light corresponds to $m = 0$ and the second time, at B, is when $m = 1$. Therefore, to find the thickness at this point we can substitute and solve the above equation.

$$2t_B = \left(1 + \frac{1}{2}\right)\frac{\lambda_y}{n_o}$$

$$t_B = \left(\frac{3}{4}\right)\frac{580}{1.5}$$

$$t_B = 290\ nm$$

The next question asks us to prove that an oil thickness of $0.2t_B$ corresponds to no visible light being reflected. We start by rearranging the first equation to solve for the wavelength.

$$\lambda = \frac{n_o 2t}{\left(m + \frac{1}{2}\right)}$$

This equation expresses the wavelength of light that is reflected back as a function of the thickness, t, and the order, m. At the point A we can write the equation as a function of m.

$$\lambda(m) = \frac{1.5 \cdot 2 \cdot 0.2 \cdot 290}{\left(m + \frac{1}{2}\right)} = \frac{174}{\left(m + \frac{1}{2}\right)}$$

When $m = 0$ we have

$$\lambda(0) = \frac{174}{\left(0 + \frac{1}{2}\right)} = 348\ nm$$

Which is in the ultraviolet region, i.e. not visible. For all other values of m greater than or equal to 1, the denominator is greater than 1 and therefore the wavelength will be less than $174\ nm$, which will continue to be outside of the visible light region. Therefore, no visible light will be reflected at this thickness and a dark region will appear.

Final Summary for Wave Nature of Light – Interference

Features of the Wave Nature of Light

The two key observations that strongly support the wave theory of light are:

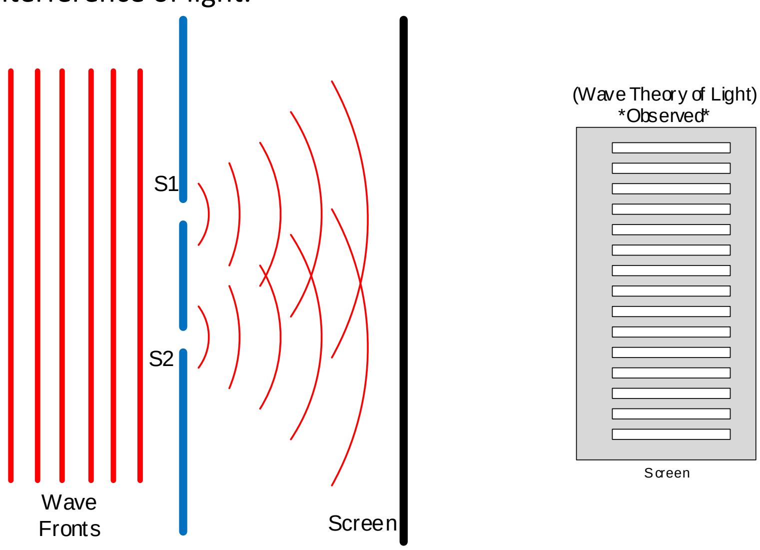
Interference: When two waves add they form a resultant wave of greater, lower, or same amplitude.

- *Constructive Interference*: Two waves come together that are completely in phase, i.e. crests and throughs align, so that the resulting amplitude is greater than the two individual waves. The amplitude will double when the two waves have the same amplitude.
- *Destructive Interference*: Two waves come together that are completely out of phase, i.e. crest of one aligns with the through of the other, so that the resulting amplitude is less than the two individual waves. The amplitude will be zero when the two waves have the same amplitude.

Diffraction: The bending of waves around an obstacle.

Young's Double-Slit Experiment

Young's double-slit experiment clearly shows the wave nature of light by demonstrating both diffraction and interference of light.



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Young's Double-Slit Wave Interference Small Angle Approximation Relationships	
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Destructive Interference (Dark Spots)	$\theta_m = (m + 1/2)\frac{\lambda}{d}, \quad y_m = (m + 1/2)\frac{L\lambda}{d}$

Interference in Thin Films

Interference can occur when light is reflected from the top and bottom surface of a thin film.

The light reflected from the bottom surface travels an extra distance that is related to the thickness of the film and the viewing angle.

Two additional phenomena need to be considered when trying to predict the interference patterns in thin films.

1. A phase change as described below.

Reflecting surface n_1 to n_2	*Phase of Reflected Wave*
$n_2 > n_1$	Phase change of $\lambda/2$
$n_2 < n_1$	Phase change of 0

2. When light travels from one medium to the other the frequency remains unchanged, but the wavelengths are related as follows:

$$\frac{\lambda_{n_2}}{\lambda_{n_1}} = \frac{n_1}{n_2} \rightarrow \lambda_{n_2} = \left(\frac{n_1}{n_2}\right)\lambda_{n_1}$$

When the first medium is a vacuum, i.e. $n_1 = 1.0$, we have

$$\lambda_{n_2} = \frac{\lambda}{n_2}$$

By: ferrantetutoring