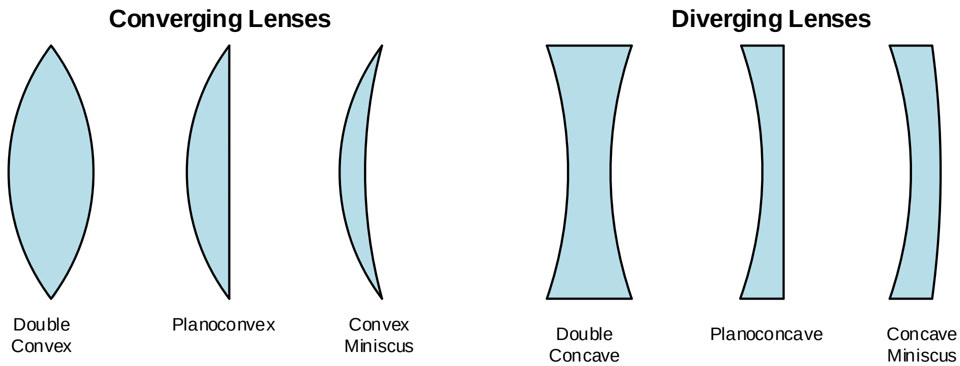
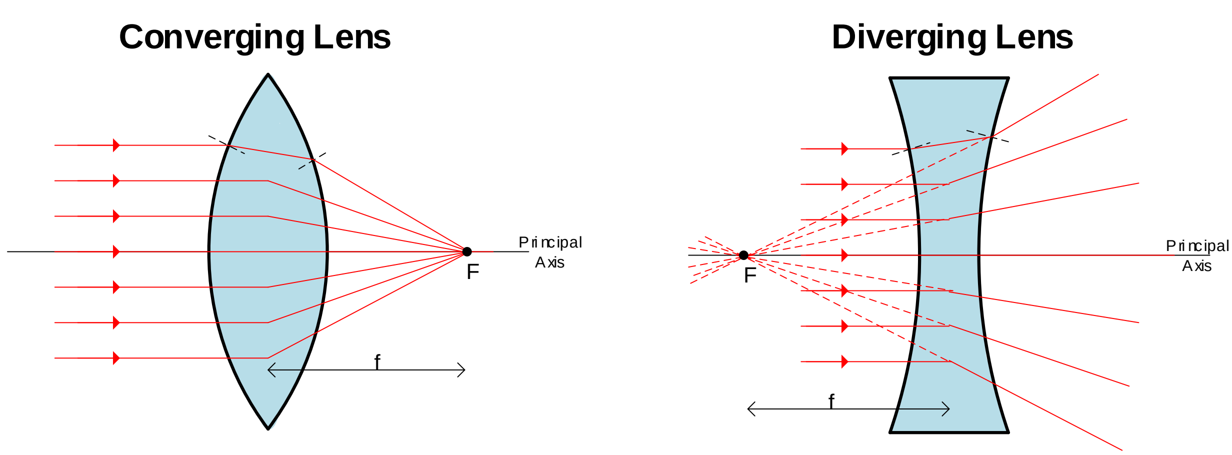
Geometric Optics – Lenses

In the previous lesson we saw that curved mirrors can be used to alter the image of an object. The altered image is produced by the reflection of light rays from the surface of the mirror. Lenses, a material for which light is *refracted*, can also be used to alter the image of objects. Lenses are one of the one of most important optical devices and are used in many specialized instruments, e.g. eyeglasses, magnifying glasses, cameras, microscopes, telescopes, etc. A thin lens is usually circular in cross section and its two faces are portions of a sphere. The two faces can be concave, convex, or plane. The two categories of lenses we will discuss in this lesson, converging and diverging, are shown below.



We will generally imagine the first type in each of the above categories, i.e. double convex and double concave, but the others behave similarly.

Just as with mirrors, lenses are primarily defined by their focal point. As shown below, parallel rays are refracted through a converging lens and, according to Snell's law, are brought together to a point on the principal axis, i.e. the focal point. Conversely, parallel rays are refracted through a diverging lens and, according to Snell's law, diverge away from each other. Similar to convex mirrors, if we trace these diverging rays backwards, they are brought together on the original side of the lens to a point on the principal axis, i.e. the focal point. The direction of the refracted rays can be surmised by assuming the index of refraction of the lens is greater than that of the air. You should perform a simple exercise by drawing a normal at a point on each side of the lens and observe the direction of the refracted ray at that point.

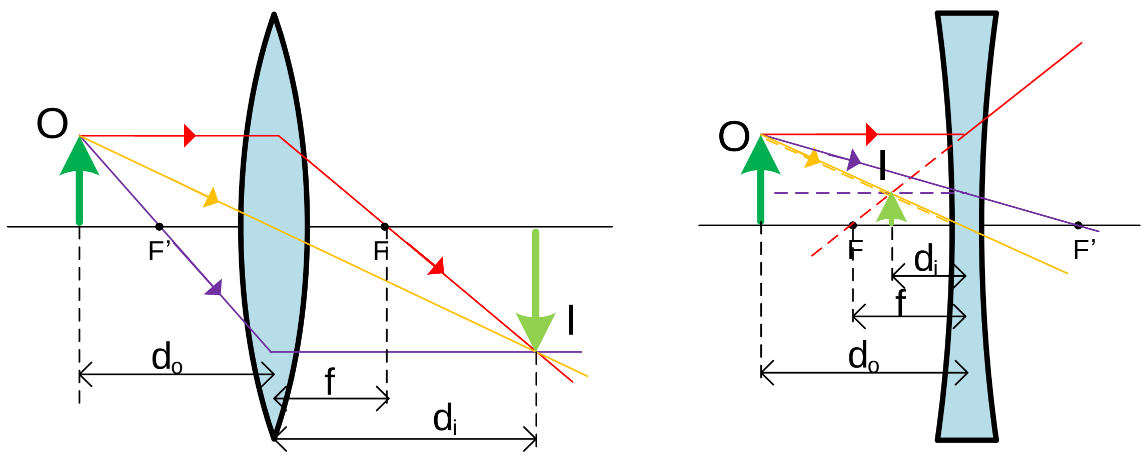


Extended Objects:

Ray tracing, as was used for mirrors, is also used to understand the images produced from looking at objects through lenses. The same three principal rays that were used for mirrors are used for lenses.

- Ray 1: Is drawn parallel to the axis and therefore passes through F upon reflection.
- Ray 2: Is drawn passing through F and therefore is parallel to the axis upon reflection.
- Ray 3: Is drawn passing through the center of the lens and therefore reflects back along the same path.

The figures below illustrate the procedure for a converging and diverging lens.



As was mentioned in the mirror lesson high accuracy is difficult using hand drawn ray tracing. Fortunately, the mirror equations we used in the previous lesson also apply to lenses. However, it is important that we use the sign conventions as described below.

Lens Equations (Same as Mirror Equations)	
$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$	$m = \frac{h_i}{h_0} = -\frac{d_i}{d_0}$ m: represents the lateral magnification
Conventions:	
• The focal length, f, is positive for converging lenses and negative for diverging lenses. • Object height, h_o, and image height, h_i, are positive if the image is upright and negative if inverted. • Object distance, d_o, is positive if it is on the side of the lens from which the light is coming, otherwise it is negative. • The image distance, d_i, is positive if it is on the opposite side of the lens from which the light is coming, otherwise it is negative. • The image is considered a *real image* if light rays pass through it and a *virtual image* is the light rays do not pass through it.	

Example 1: A stamp collector uses a converging lens with a focal length of 24 cm to view a stamp 18 cm in front of the lens.

a.) Determine the position and magnification of the image analytically.
b.) Draw a ray diagram to locate the approximate location and size of the image.

Solution: For part a. we directly use the lens equations. According to the sign convention both the focal length and the object distance are taken as positive.

The image distance can be found as follows:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$d_i = \left(\frac{1}{24} - \frac{1}{18}\right)^{-1} = -72\ cm$$

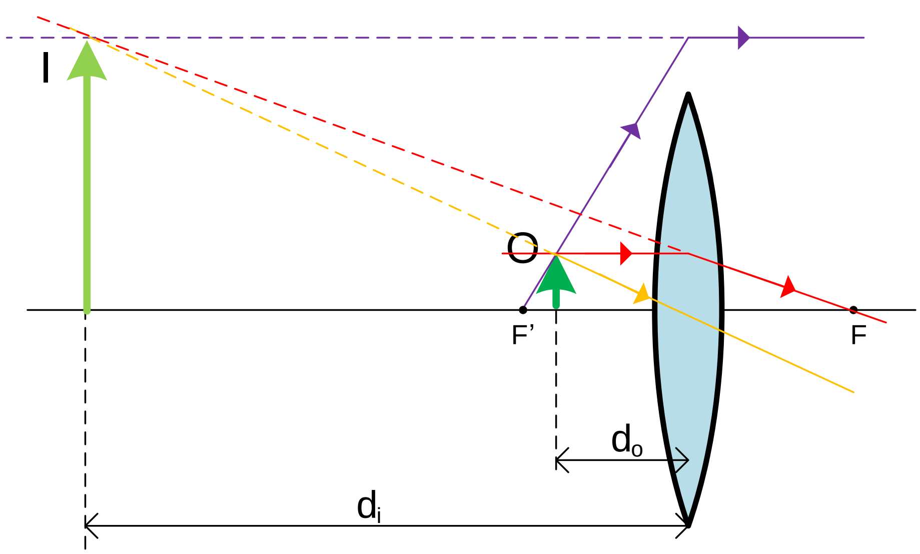
Which indicates the image is same side of the lens as the object.

Next, the magnification is found as follows

$$m = -\frac{d_i}{d_0} = -\frac{-72}{18} = 4$$

Which indicates that the image is upright.

For part b. we sketch a ray diagram to illustrate this scenario.



Example 2: A 50 cm high object is 100 cm away. What focal length is required for a diverging lens so that the image appears 20 cm from the lens? What is the height of the resulting image?

Solution: With a converging lens the object and image both appear on the side of the lens from which the light is coming. Therefore, according to the sign convention, the object distance is positive, and the image distance is negative. The focal length is then found as follows:

$$f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1}$$

$$f = \left(\frac{1}{100} + \frac{1}{-20}\right)^{-1}$$

$$f = -25\ cm$$

Note the focal length is negative, which we expect for a diverging lens.

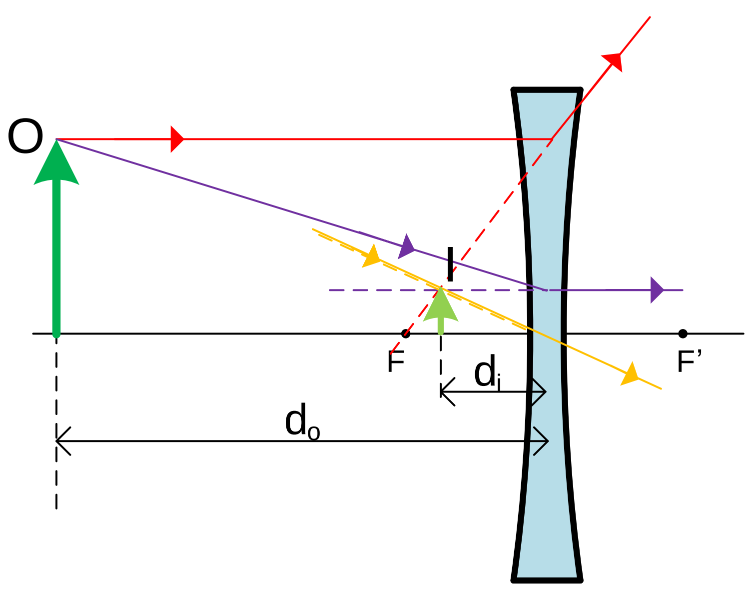
The height of the resulting image is found by first finding the lateral magnification.

$$m = -\frac{d_i}{d_0} = -\frac{-20}{100} = 0.2$$

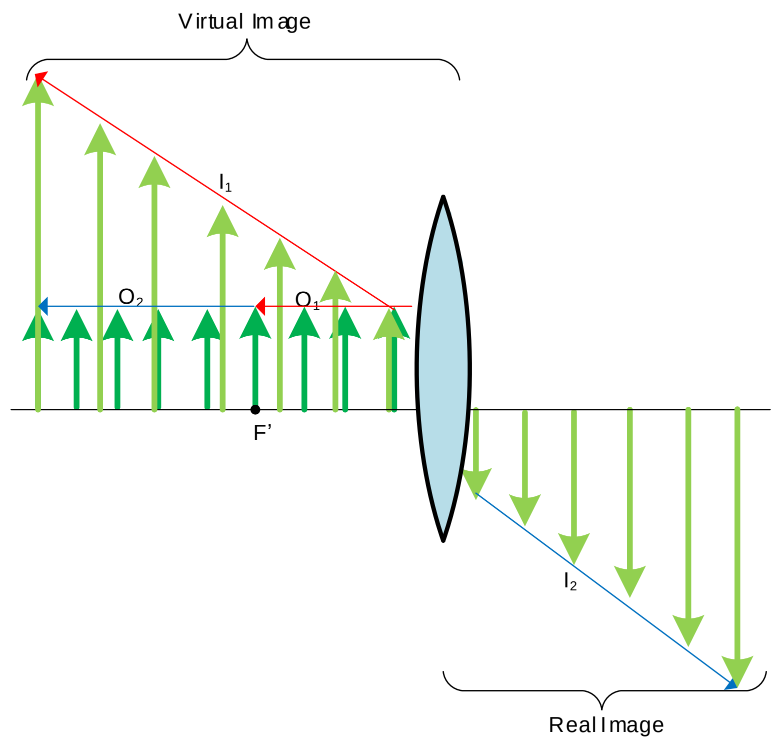
Therefore, the image is smaller and upright, with a height, h_i as shown.

$$h_i = 0.2h_i = 0.2(50) = 10\ cm$$

The ray diagram is shown below for illustration.



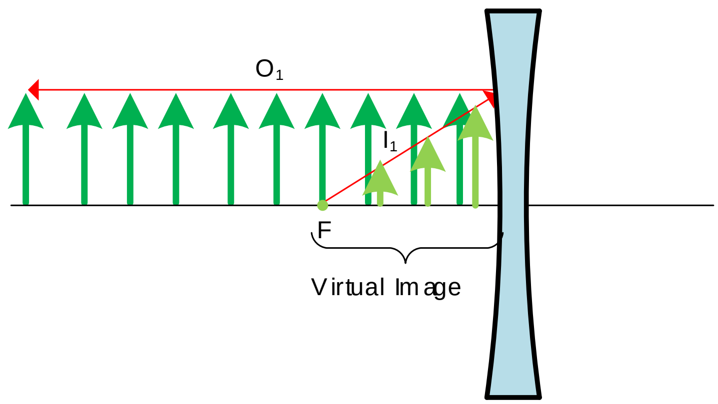
As we did for mirrors, we show below how the image location and size changes as a function of the object distance. We start with the converging lens shown below.



The illustration above shows the object and image location moving over two regions, which are explained below.

- $O_1, I_1, (0 < d_o < f)$:
  - As the object moves away from the lens towards the focal point, a *virtual* image is formed that moves further away from the mirror and gets larger.
- $(d_o = f)$:
  - As the object continues to move away from the lens and approaches the focal point the virtual image approaches a distance of infinity with height approaching infinity. Note if the object were approaching the focal point from the other side an inverted *real* image would be approaching a distance of infinity with a height approaching infinity.
- $O_2, I_2, (f < d_o < \infty)$:
  - As the object moves from the focal point to infinity an inverted *real* image begins to move in from a distance of infinity and begins to decrease in size.

The next figure shows how the image varies as a function of the object distance for a diverging lens.



The illustration shows the object and image location moving over a single region, which is explained below.

- $O_1, I_1, (\, 0 < d_o < \infty)$:
  - As the object moves away from the mirror towards positive infinity, a *virtual* image is formed that moves further away from the mirror and gets smaller. Note that the image starts with the same height as the object and approaches zero in height at the focal point when the object is at infinity.

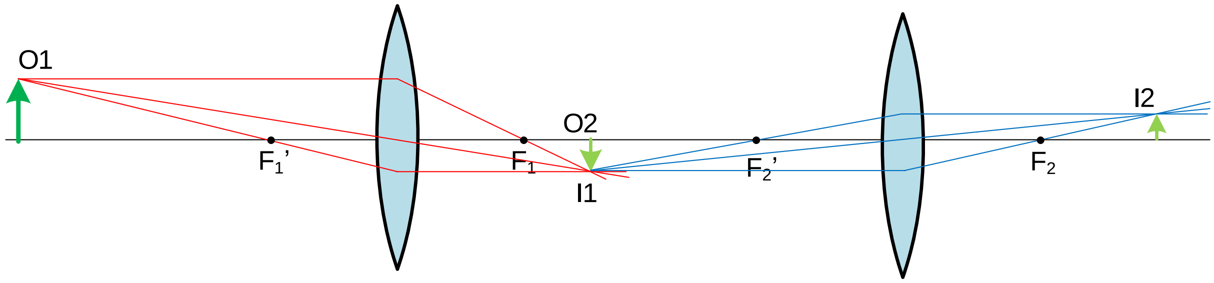
In both cases you should go back and see the similarity between these and the mirror figures.

Combination of Lenses:

As we will soon see lenses are used to make many different optical instruments. Most of the instruments use more than one lens to produce the desired image. Just as we saw with multiple mirrors the image from a first lens will act as the object for a second lens, and so on. Furthermore, as we will see in the next example, the total magnification will be the product of the magnifications of the individual lenses.

Example 3: Two converging lenses, with focal lengths $f_1 = 20\ cm$ and $f_2 = 25\ cm$, are placed 80 cm apart as shown. An object is placed 60 cm in front of the first lens. Find the location and magnification of the image formed by the combination of lenses.

Solution: In this case we will start with a ray diagram to give us an idea of what we should expect from the equations.



We start by solving the lens equation for the image distance produced by the first lens.

$$d_{i1} = \left(\frac{1}{f_1} - \frac{1}{d_{o1}}\right)^{-1}$$
$$d_{i1} = \left(\frac{1}{20} - \frac{1}{60}\right)^{-1} = 30\ cm$$

The magnification of the first image is then

$$m_1 = -\frac{d_{i1}}{d_{o1}} = -\frac{30}{60} = -0.5$$

Next, we use the lens equation for the second. Note that the image from the first lens is the object for the second lens, which is $80 - 30 = 50\ cm$ in front of the second lens.

$$d_{i2} = \left(\frac{1}{f_2} - \frac{1}{d_{o2}}\right)^{-1}$$
$$d_{i2} = \left(\frac{1}{25} - \frac{1}{50}\right)^{-1} = 50\ cm$$

The magnification for the second image, (with respect to the first), is then

$$m_2 = -\frac{d_{i2}}{d_{o2}} = -\frac{50}{50} = -1.0$$

Which indicates the second image is inverted with respect to the first image. Note this makes the final image upright with respect to the original object, which is what we get if we multiple the two magnifications.

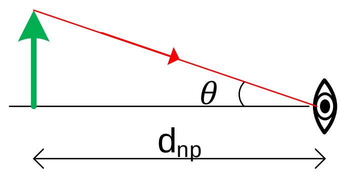
$$m = m_1 \cdot m_2 = (-0.5) \cdot (-1.0) = 0.5$$

Optical Instruments:

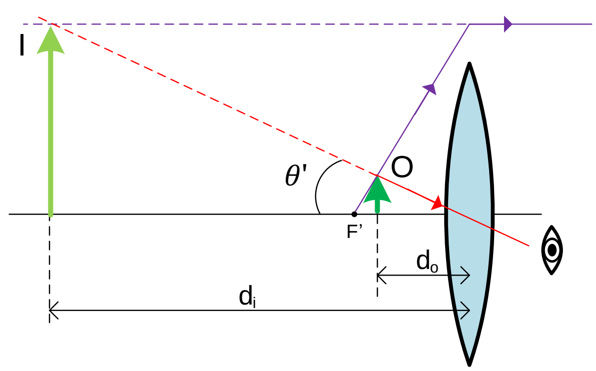
As mentioned, many optical instruments use a lens as their basic operating element. In this section we will look at the magnifying glass and the compounds microscope. Of course, there are many other important instruments such as telescopes, cameras, and eyeglasses.

Magnifying Glass:

A simple magnifier, i.e. a magnifying glass, is simply a converging lens. When we attempt to look at a small object, we usually bring it closer to our eyes. This action has the effect of creating a larger image for you to observe more detail. However, our eyes can only accommodate up to a point, called the *near point,* d_{np}, before the image becomes blurry. For most people, the near point is 25 cm, (as we age this point becomes further away).



How large the object appears can be related to the angle, θ, that is subtended by the object at the eye as shown in the figure. Although bringing the object closer than d_{np} would increase this angle, the eye will begin to have trouble with the image detail. The magnifying glass allows us to bring the object closer to our eye so that it subtends a greater angle, while preserving the detail since the image can be made to appear further away. We illustrate this below by placing the object just inside the focal point of a converging lens.



The image seen by the eye now subtends an angle, θ', and as long as $d_i \geq d_{np}$ the detail will be preserved. The magnifying power of a magnifying glass is measured in the angular domain and is given as

$$M = \frac{\theta'}{\theta}$$

Using h_o as the height of the object we can write an equation for each of the figures above

Object placed at the near point - no lens. $\tan(\theta) = \frac{h_o}{d_{np}}$	Object placed near focal point of a lens. $\tan(\theta') = \frac{h_o}{d_o}$

For small angles we can use the approximation; $tan(\theta) \approx \theta$.

$$M = \frac{\theta'}{\theta} = \frac{h_o/d_o}{h_o/d_{np}} = \frac{d_{np}}{d_o}$$

If we place the object at the focal point of the lens, $d_o = f$, the eye is focused on an image at infinity and the magnification is given as

$M = \frac{d_{np}}{f}$	Eye focused on image at ∞

However, the magnification of a lens can be slightly increased by instead having the eye focus on an image at the near point, d_{np}. In this case to write the magnification as a function of the focal length we solve the lens equation for $1/d_o$ with $d_i = -d_{np}$ and substitute as shown below

$$\frac{1}{d_o} = \frac{1}{f} + \frac{1}{d_{np}}$$

Therefore**,**

$$M = \frac{d_{np}}{d_o} = d_{np}\left(\frac{1}{f} + \frac{1}{d_{np}}\right) = \frac{d_{np}}{f} + 1$$

$M = \frac{d_{np}}{f} + 1$	Eye focused on image at d_{np}

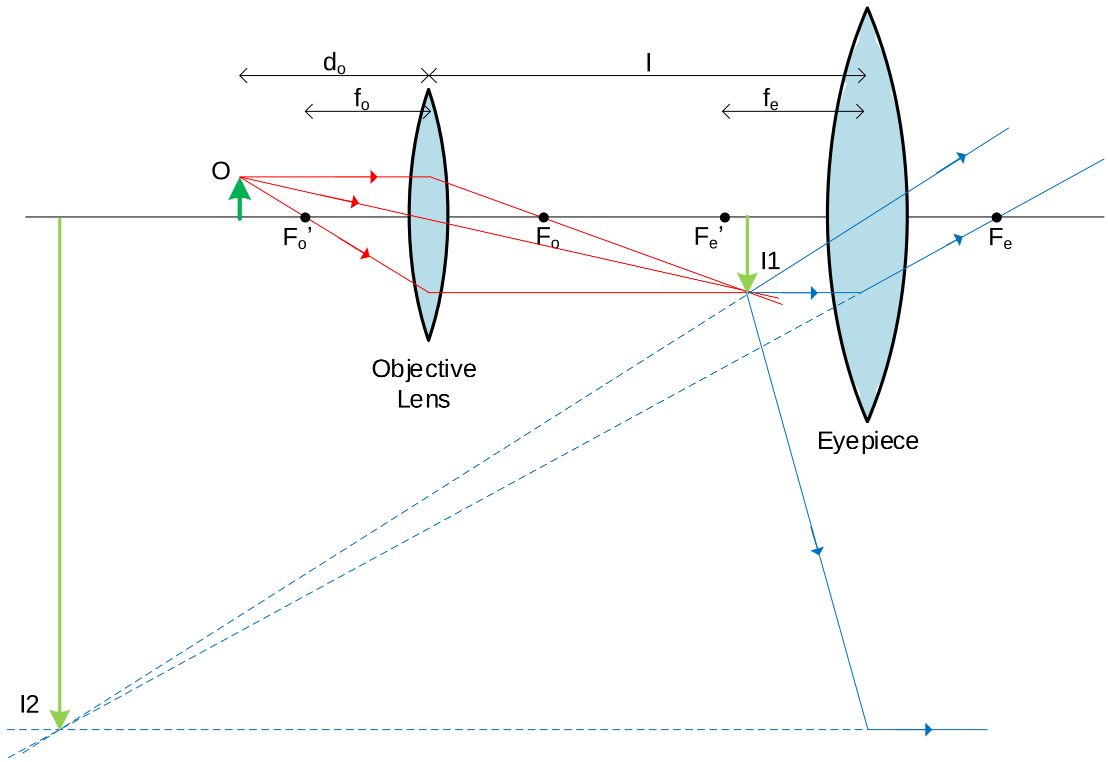
Example 5: For most people, our near point increases as we age. Compare the magnification for a simple magnifier with a focal length of 8 cm for a younger person whose near point distance is 20 cm to an older person, whose near point distanced is 30 cm.

Solution: Using the second formula where the eye is focused at the near point, we have

Younger Person	$M = \frac{d_{np}}{f} + 1 = \frac{20}{8} + 1 = 3.5$
Older Person	$M = \frac{d_{np}}{f} + 1 = \frac{30}{8} + 1 = 4.75$

Compound Microscope:

The compound microscope uses a two lens system. The first lens, called the objective lens, is where light from the object first enters. The second lens, called the eyepiece, uses the image produced by the objective lens as the object for which it will form an image. The figure shown below illustrates how the compound microscope works.



The object to be observed is placed just beyond the focal point of the objective lens. The first image, $I1$, is an inverted real image that is enlarged and quite far from the objective lens. The distance between the lenses, l, is such that the first image falls just inside the focal point of the eyepiece. The eyepiece then act as a simple magnifier and further enlarges the object. The second image, $I2$, is seen by the observer through the eyepiece.

The overall magnification of the object is the product of the magnification of the two lenses. The magnitude of the magnification of the first lens is given as

$$m_o = \frac{d_i}{d_o} = \frac{l - f_e}{d_o}$$

Where, we assumed d_i is at at the eyepiece focal point. As mentioned, the eyepiece acts as a simple magnifier, and since we assumed the object, (image from objective lens), is right at the focal point, we can use the ‘eye focused at infinity’ formula for the angular magnification.

$$M_e = \frac{d_{np}}{f_e}$$

The overall magnification, M, can be measured as the product of the lateral magnification of the objective lens and the angular magnification of the eyepiece.

$$M = M_e m_o = \frac{d_{np}}{f_e}\left(\frac{l - f_e}{d_o}\right)$$

When the eyepiece focal length, f_e, is small compared to l we can approximate $l - f_e \approx l$. Furthermore, if we place the object close to the focal point of the objective lens, we can approximate $d_o \approx f_o$. Using these approximations, we can write the magnification of a compound microscope as a function of the focal lengths and the distance between the lenses.

$$M = \frac{d_{np} l}{f_e f_o}$$

Example 6: A compound microscope has a 1.8 cm focal length eyepiece and an 0.80 cm focal length objective lens. Assuming the eye is focused at infinity calculate (a) the position of the object if the distance between the lens is 16.0 cm, and (b) the total magnification.

Solution: Since the eye is focused at infinity, we assume the image from the objective lens is at the focal point of the eyepiece. Therefore, we can use the lens equation to solve for the object distance with respect to the objective lens.

$$\begin{aligned} d_o &= \left(\frac{1}{f_o} - \frac{1}{d_i}\right)^{-1} \\ &= \left(\frac{1}{f_o} - \frac{1}{(l - f_e)}\right)^{-1} \\ &= \left(\frac{1}{0.8} - \frac{1}{(16 - 1.8)}\right)^{-1} \cong 0.85\ cm \end{aligned}$$

For the magnification we can use the first equation from above, (before the approximation), with $d_{np} = 25\ cm$

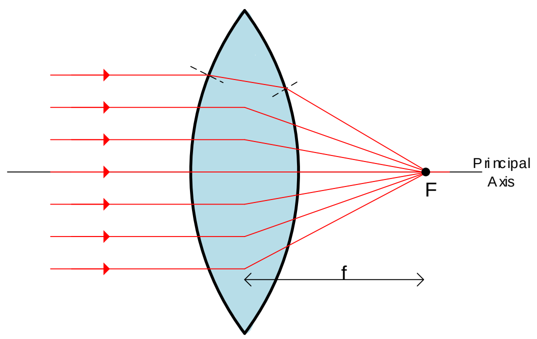
$$\begin{aligned} M &= \frac{d_{np}}{f_e}\left(\frac{l - f_e}{d_o}\right) \\ &= \frac{25}{1.8}\left(\frac{16 - 1.8}{0.85}\right) \cong 232 \end{aligned}$$

Final Summary for Geometric Optics – Lenses

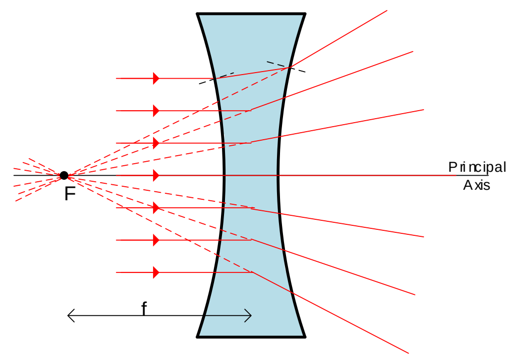
Lenses

Light rays refract inward when impinging on a converging lens and outward when impinging on a diverging lens.

Converging Lens

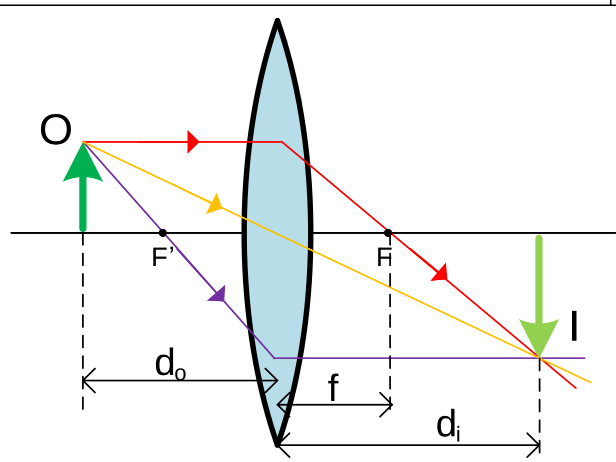


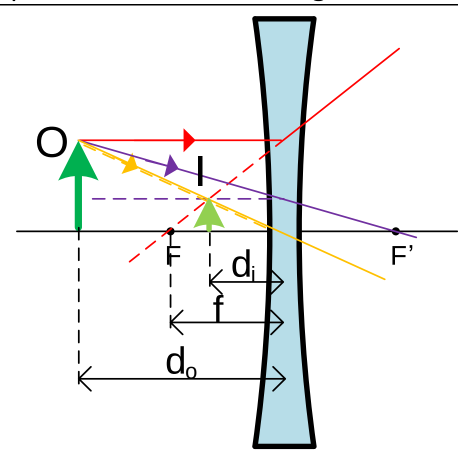
Diverging Lens



Lens Equations (Same as Mirror Equations)

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$	$$m = \frac{h_i}{h_0} = -\frac{d_i}{d_0}$$ m: represents the lateral magnification





Conventions:

- The focal length, f, is positive for converging lenses and negative for diverging lenses.
- Object height, h_o, and image height, h_i, are positive if the image is upright and negative if inverted.
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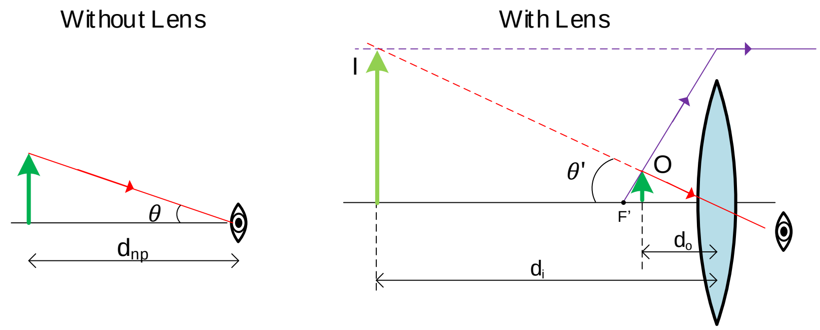
- The image distance, d_i, is positive if it is on the opposite side of the lens from which the light is coming, otherwise it is negative.
- The image is considered a *real image* if light rays pass through it and a *virtual image* is the light rays do not pass through it.

Simple Magnifier

A Simple Magnifier is made of a converging lens.



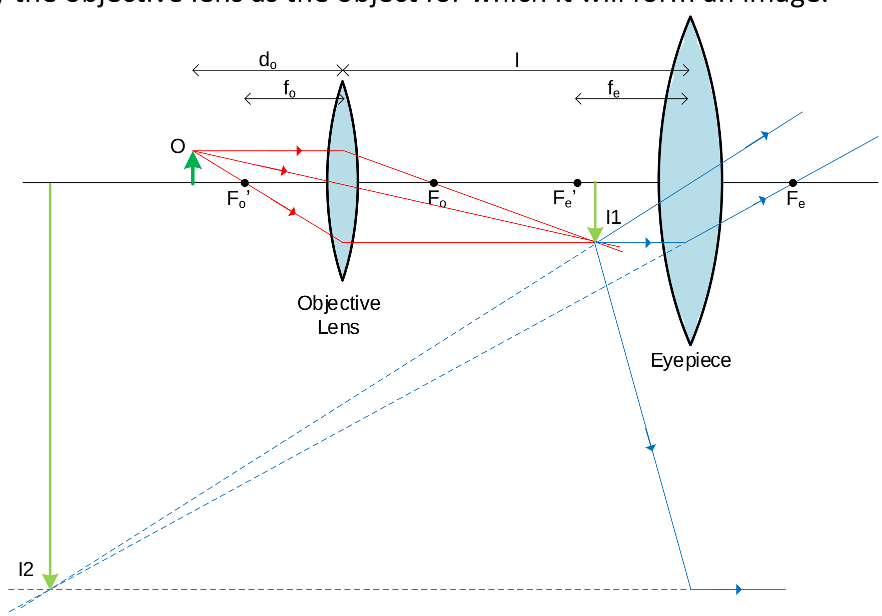
The angular magnification for a simple magnifier is given as

$M = \frac{d_{np}}{f}$	Eye focused on image at ∞
$M = \frac{d_{np}}{f} + 1$	Eye focused on image at d_{np}

Where $d_{np} = 25\ cm$ for the normal eye

Compound Microscope

The compound microscope uses a two lens system. The first lens, called the objective lens, is where light from the object first enters. The second lens, called the eyepiece, uses the image produced by the objective lens as the object for which it will form an image.



The magnification, M, is given as

$$M = \frac{d_{np}}{f_e}\left(\frac{l - f_e}{d_o}\right)$$

Using the approximations $l - f_e \approx l$, and $d_o \approx f_o$ we can also write the equation as

$$M = \frac{d_{np} l}{f_e f_o}$$

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