Limits – Basic Limit Laws

In the previous section we evaluated limits using numerical and graphing techniques. These techniques helped to provide us with an intuitive understanding of limits, however they are usually not the most efficient ways to evaluate limits. In the upcoming sections we will present more rigorous methods for evaluating limits. The set of limit laws presented in this section will set a foundation for us to be able to evaluate limits using these more rigorous methods. We state the laws below without proof and then give a few examples to make sure we know how to properly use them.

Basic Limit Laws
If $\lim_{x\to c}\{f(x)\}$ and $\lim_{x\to c}\{g(x)\}$ exist, then the following laws apply
Sum and Difference Law $$\lim_{x\to c}\{f(x) \mp g(x)\} = \lim_{x\to c}\{f(x)\} \mp \lim_{x\to c}\{g(x)\}$$
Constant Multiple Law $$\lim_{x\to c}\{Kf(x)\} = K\lim_{x\to c}\{f(x)\}$$
Product Law $$\lim_{x\to c}\{f(x)\cdot g(x)\} = \left(\lim_{x\to c}\{f(x)\}\right)\cdot\left(\lim_{x\to c}\{g(x)\}\right)$$
Quotient Law $$\lim_{x\to c}\left\{\frac{f(x)}{g(x)}\right\} = \frac{\lim_{x\to c}\{f(x)\}}{\lim_{x\to c}\{g(x)\}}$$ Provided, $\lim_{x\to c}\{g(x)\} \neq 0$
Power Law $$\lim_{x\to c}\left\{\left(f(x)\right)^n\right\} = \left(\lim_{x\to c}\{f(x)\}\right)^n$$ Where, n is a positive integer
Root Law $$\lim_{x\to c}\left\{\sqrt[n]{f(x)}\right\} = \sqrt[n]{\lim_{x\to c}\{f(x)\}}$$ Where, if n is even we assume $\lim_{x\to c}\{f(x)\} \geq 0$

The examples below show how these laws can be used.

1. Evaluate the following two limits:
   a.) $\lim_{x\to 2}\{(x+1)(3x^2-9)\}$
   b.) $\lim_{x\to 25}\left\{\frac{3\sqrt{x}-\frac{1}{5}x}{(x-20)^2}\right\}$

$= \lim_{x\to 2}\{(x+1)(3x^2-9)\}$ $= \lim_{x\to 2}\{(x+1)\}\lim_{x\to 2}\{(3x^2-9)\}$ $= \left(\lim_{x\to 2}\{x\} + \lim_{x\to 2}\{1\}\right)\left(3\left(\lim_{x\to 2}\{x\}\right)^2 - \lim_{x\to 2}\{9\}\right)$ $= (2+1)(3(2)^2-9)$ $= (3)(12-9)$ $= 9$	$= \lim_{x\to 25}\left\{\frac{3\sqrt{x}-\frac{1}{5}x}{(x-20)^2}\right\}$ $= \frac{\lim_{x\to 25}\left\{3\sqrt{x}-\frac{1}{5}x\right\}}{\lim_{x\to 25}\{(x-20)^2\}}$ $= \frac{3\sqrt{\lim_{x\to 25}\{x\}}-\frac{1}{5}\lim_{x\to 25}\{x\}}{\left(\lim_{x\to 25}\{x\}-\lim_{x\to 25}\{20\}\right)^2}$ $= \frac{3\sqrt{25}-\frac{1}{5}\cdot 25}{(25-20)^2}$ $= \frac{2}{5}$

2. Evaluate $\lim_{x\to 0}\left\{\frac{\sin(x)}{x}\right\}$

Recall we had previously evaluated this limit numerically and found the value to be 1.0. Let's see if we can apply the limit laws and get the same answer.

$$= \lim_{x\to 0}\left\{\frac{\sin(x)}{x}\right\}$$

$$= \frac{\lim_{x\to 0}\{\sin(x)\}}{\lim_{x\to 0}\{x\}}$$

$$= \frac{0}{0}$$

This result would lead us to believe that the limit doesn't exist. However, applying the quotient law requires that the limit of the denominator is not equal to 0, and therefore we cannot apply it in this case.

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