# Pre-Calculus Functions Review

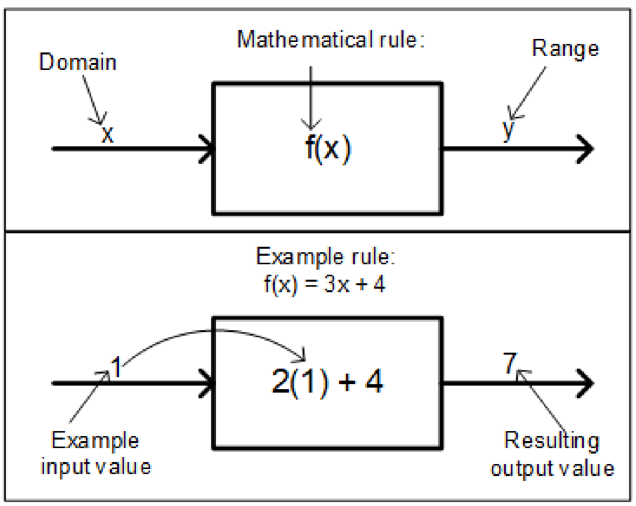
Functions are one of the most important concepts in calculus. To understand calculus, it is critical to have a fundamental understanding of what functions are and how to work with them. Let's start with a formal definition of a function.

A ***function*** is a mathematical rule that assigns a *unique* output value to every input value.

- The set of allowable input values, usually denoted by $x$, is called the *domain* of the function.
- The set of all output values, usually denoted by $y$, is called the *range* of the function.
- The function itself is usually denoted by $f$.
- Applying an input value, $x$, to a function $f$, to produce an output value is denoted as:

$$y = f(x)$$

We can also think of a function as a machine produces an output value for every input value in its domain. The figure below is useful to keep in mind when using functions.



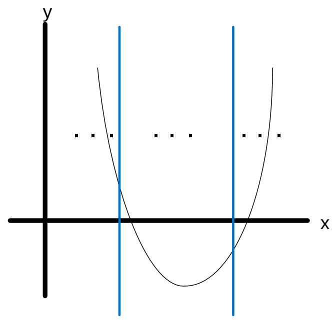

For now, we focus on real valued functions only, which means all input values and output values must be real numbers. Some example functions are listed below:

| | | | |
|---|---|---|---|
| $f(x) = 3x + 4$ | $f(x) = \frac{1}{5}x^2 + 4x + 1$ | $f(x) = \frac{1}{x}$ | $f(x) = \sqrt{x}$ |

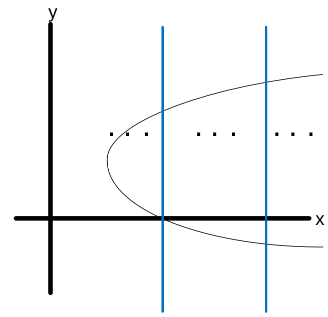
Finally, note that in the formal definition of a function we said that it "assigns a ***unique*** output value to every input value". The term *unique* is important and if not satisfied the relationship is not formally considered a function. In plain terms when we input a value into our function machine, we expect one output and we expect that output to be the same each time we use the same input value. Graphically, we can tell if a relationship is a function by the so-called *Vertical Line Test*.

*Vertical Line Test:* We imagine vertical lines being drawn for all $x$ values on a graph of a curve. If any such line passes through the curve more than once the relationship is not a function

***Function***




***Not a Function***




**Basic Function Types (Linear, Quadratic, Piecewise):**

*Linear Functions:* A linear function is one in which the graph is a straight line. Algebraically, linear functions are usually expressed in one of three different forms shown below.

| | | |
|---|---|---|
| **Slope Intercept Form** | $y = mx + b$ | $m$= slope, $b$ = $y$-intercept |
| **Point Slope Form** | $(y - y_1) = m(x - x_1)$ | $m$= slope, $(x_1, y_1)$ is a point on the line. |
| **Standard Form** | $Ax + By = C$ | $A, B$, and $C$ are constants |

The concept of the slope, labeled as $m$ above, is fundamental to the study of calculus. For a linear equation the slope is a measure of the *steepness* of the line. We measure it by taking any two points on a line and computing the ratio of the change in $y$ to the change in $x$.

$$m = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

As an example, let's find the equation of the line for the graph below. We can start by finding the slope. Recall for linear equations you can use any two points because the slope is the same everywhere on the line.

***Using $P_1$ and $P_2$***

$$m = \frac{\Delta y}{\Delta x} = \frac{(7-3)}{(3-1)} = \frac{4}{2} = 2$$

***Using $P_3$ and $P_4$***

$$m = \frac{\Delta y}{\Delta x} = \frac{((-1)-(-3))}{((-1)-(-2))} = \frac{2}{1} = 2$$

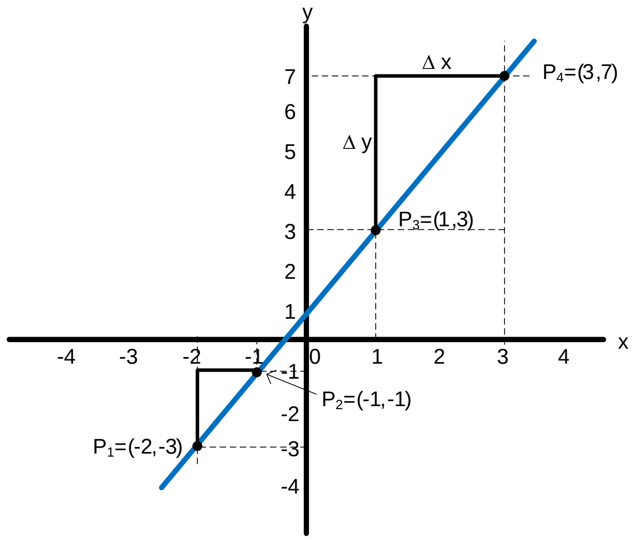
Now we can use the point-slope formula. Note, once we find the slope, we can use any known point on the line in the formula below.

***Using $P_1$***

$$\begin{aligned}(y - (-3)) &= 2(x - (-2)) \\ (y + 3) &= 2(x + 2) \\ y &= 2x + 4 - 3 \\ \boldsymbol{y} &= \mathbf{2}\boldsymbol{x} + \mathbf{1}\end{aligned}$$

***Using $P_4$***

$$\begin{aligned}(y - 7) &= 2(x - 3) \\ y &= 2x - 6 + 7 \\ \boldsymbol{y} &= \mathbf{2}\boldsymbol{x} + \mathbf{1}\end{aligned}$$



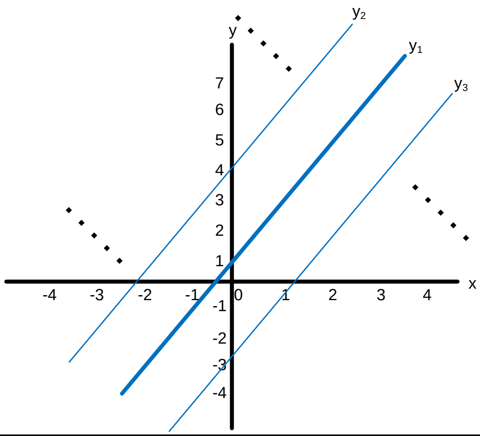
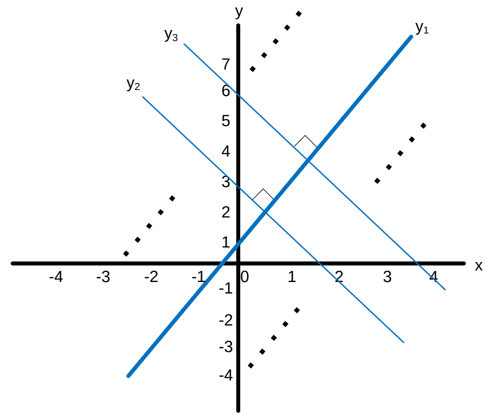

For non-linear functions the steepness of the curve is different for different locations along the curve. Therefore, in this case the slope is not a constant value but rather is a function of $x$. It is indeed possible to define it as such, and this concept is the first of the two main concept we will learn while studying calculus.

*Parallel and Perpendicular Lines*

Recall that there exists a fixed relationship between the slopes of a set of parallel as well as perpendicular lines.

- *Parallel lines* have the same slope, i.e. $m_2 = m_1$
- *Perpendicular lines* have slopes that are the negative inverses of each other, i.e. $m_2 = -\frac{1}{m_1}$

As an example, let's take the line from above, $y = 2x + 1$, and find parallel and perpendicular lines. The figures below show that for any line there exists an infinite number of both parallel and perpendicular lines. Using the slope of the original line, $m_1 = 2$, we know the slope of the parallel or perpendicular lines from the above and need only to find the appropriate $y$-intercepts. Once these two values are known we can use the slope-intercept from to write the equations for the new lines as shown below.

| ***Parallel Lines*** | ***Perpendicular Lines*** |
|---|---|
|  |  |
| $m_1 = 2$, therefore, $m_2 = m_3 = 2$.<br><br>Using the slope-intercept form we need only to find the $y$-intercept of the parallel lines.<br><br>$y_2 = m_2x + b_2$    $y_3 = m_3x + b_3$<br>$y_2 = 2x + 4$    $y_3 = 2x - 3$ | $m_1 = 2$, therefore, $m_2 = m_3 = -1/2$.<br><br>Using the slope-intercept form we need only to find the $y$-intercept of the perpendicular lines.<br>$y_2 = m_2x + b_2$    $y_3 = m_3x + b_3$<br>$y_2 = -\frac{1}{2}x + 3$    $y_3 = -\frac{1}{2}x + 6$ |

*Quadratic Functions:*

A polynomial function is a function of the following form.

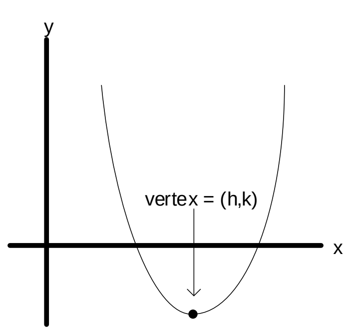
$$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x^1 + a_0$$

Where, the $a_n$'s are constants referred to as coefficients, and $n$ is the degree of the polynomial.

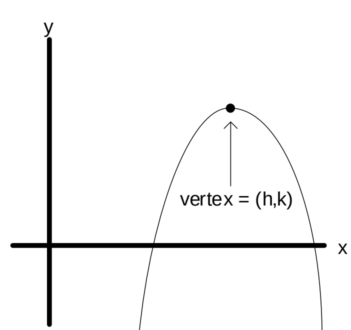
A quadratic function is a polynomial of degree 2, which is written in so-called standard form as:

$$f(x) = ax^2 + bx + c$$

The graph of a quadratic function is a *parabola.* The parabola is concave up for $a > 0$, and concave down for $a < 0$. The point, shown in the figure as $(h, k)$, is called the vertex and represents the minimum or maximum value of the function.




a>0,
concave up,
vertex is minimum




a<0,
concave down,
vertex is maximum

Identifying the vertex in a quadradic equation is often useful. The vertex is instantly identifiable when the function is written in so-called vertex form.

$$f(x) = a(x-h)^2 + k$$

Where, the point $(h, k)$ is the vertex.

A quadratic function in standard form can be converted to vertex form using the technique of *completing the squares.*

Starting with the standard form we first divide through by $a$.

$$f(x) = ax^2 + bx + c$$

$$\frac{f(x)}{a} = \left[x^2 + \left(\frac{b}{a}\right)x\right] + \left(\frac{c}{a}\right)$$

Focusing on the first two terms, we recall that a perfect square is given as:

$$(x \pm A)^2 = x^2 + 2Ax + A^2.$$

Next if we set $2A = \frac{b}{a}$, so that $A = \frac{b}{2a}$ we can rewrite the first two term as follows:

$$\left[x^2 + \left(\frac{b}{a}\right)x\right] = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$$

Substituting we can solve for the vertex form:

$$\frac{f(x)}{a} = \left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + \left(\frac{c}{a}\right)$$

$$\frac{f(x)}{a} = \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \left(\frac{4ac}{4a^2}\right)$$

$$\frac{f(x)}{a} = \left(x - \left(-\frac{b}{2a}\right)\right)^2 + \left(\frac{4ac - b^2}{4a^2}\right)$$

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \left(\frac{4ac - b^2}{4a}\right)$$

Where,

$$h = -\frac{b}{2a}$$

$$k = \frac{4ac - b^2}{4a}$$

Which is more commonly computed as:

$$k = f\left(-\frac{b}{2a}\right)$$

When working with a quadratic function its often required to set the equation to zero and solve for $x$. We can always try to factor the equation; however, we should also be familiar with the so-called *quadratic formula*, which can be derived starting with the last equation from above.

$$a\left(x-\left(-\frac{b}{2a}\right)\right)^2+\left(\frac{4ac-b^2}{4a}\right)=0$$

$$a\left(x+\frac{b}{2a}\right)^2=\left(\frac{b^2-4ac}{4a}\right)$$

$$\left(x+\frac{b}{2a}\right)^2=\left(\frac{b^2-4ac}{4a^2}\right)$$

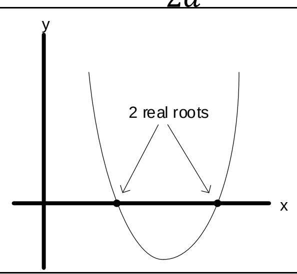
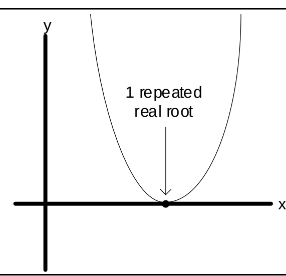
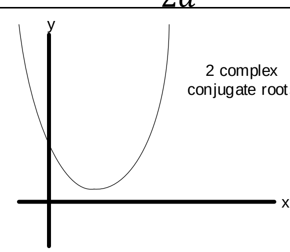
$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=\frac{\pm\sqrt{b^2-4ac}}{2a}-\frac{b}{2a}$$

$$\boldsymbol{x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

You may also recall the term inside the square root function is called the *discriminate, D, and* can be used to describe the types of solutions; also called roots or zeros.

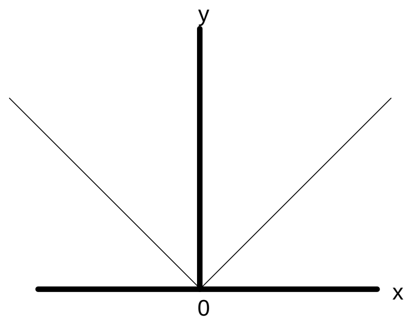
$$D=b^2-4ac$$

| $\boldsymbol{D>0}$<br>2 real roots | $\boldsymbol{D=0}$<br>1 repeated root | $\boldsymbol{D<0}$<br>2 complex conjugate roots |
|---|---|---|
| $x_{1,2}\,\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ | $x=-\frac{b}{2a}$ | $x_{1,2}\,\frac{-b\pm i\sqrt{4ac-b^2}}{2a}$ |
|  |  |  |

*Piecewise Functions:* A piecewise function is one that is defined differently for different ranges of inputs. A general definition can be given as follows:

$$f(x)=\begin{cases} f_1(x), & x<a_1 \\ f_2(x), & a_1\le x<a_2 \\ f_3(x), & a_2\le x<a_3 \\ & \vdots \\ f_n(x), & a_{n-1}\le x<a_n \end{cases}$$

One of the most widely used piecewise functions is the absolute value function, shown below:

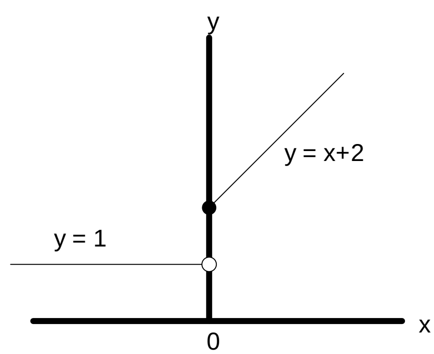



$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

For a more general case consider the following piecewise function definition.

$$f(x) = \begin{cases} 1, & x < 0 \\ x + 2, & x \geq 0 \end{cases}$$

If we plot this function, we notice that the function is not *smooth* in the sense that there is a "*jump*" in the function at $x = 0$. We will see later that this function is considered *discontinuous* at the point $x = 0$. Furthermore, notice that at $x = 0$ the function is defined by the second function, $x + 2$, hence we have a closed circle at $y = 2$ and an open circle at $y = 1$.




**Domain and Range of Functions:**

Not all functions will produce valid outputs for all inputs. Furthermore, the set of all values output from a function may not be all the real numbers. These two properties of functions can be described using the terms: *domain* and *range*, which are formally defined below.

| |
|---|
| **<u>Domain of a function, $f$:</u>**<br>The set of all possible $x$ values that when applied to the function produce a *valid* output value, $y$.<br>Note: For real valued functions a valid output is any real number. |
| **<u>Range of a function, $f$:</u>**<br>The set of all values, $y$, that are produced by the function as a result of applying all values, $x$, in the domain. |

**Examples:**

Find the domain and range of the following functions:

a. $f(x) = x^2 - 2x - 3$

b. $f(x) = \sqrt{x+5}$

c. $f(x) = \frac{1}{x^2-2x+3}$

d. $f(x) = \frac{1}{\sqrt{x^2-2x+3}}$

**Solutions:**

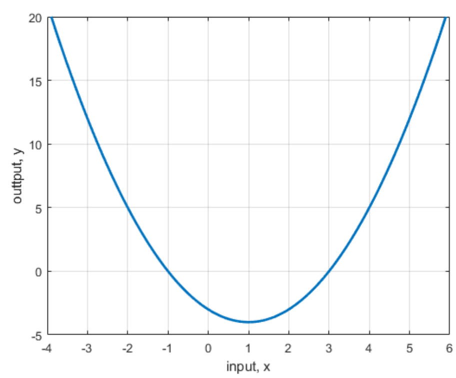
a. $f(x) = x^2 - 2x - 3$

The domain for polynomials is all real numbers. $D: (-\infty, \infty)$.

For the range we start by finding the vertex, which we can do by completing the square.

$$f(x) = [x^2 - 2x] - 3$$
$$f(x) = [(x-1)^2 - 1] - 3$$
$$f(x) = (x-1)^2 - 4$$

The vertex is at the point, $P = (1, -4)$. Furthermore, since the coefficient for the highest degree term is positive, the parabola is concave up with a minimum value at $y = -4$. Therefore, the range is: $R: [-4, \infty)$. See the graph below for additional insight.




$D: (-\infty, \infty)$

$R: [-4, \infty)$

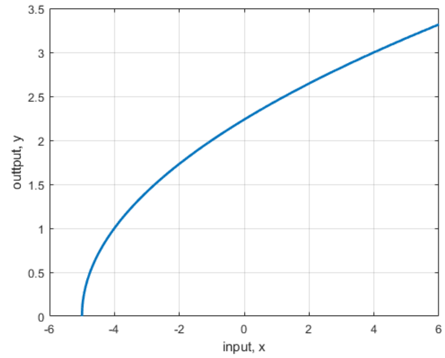
b. $f(x) = \sqrt{x+5}$

The square root of negative number results in an imaginary number, and since we are considering only real valued functions it is not a valid output. The domain of the above function can then be found as follows.

$$x + 5 \geq 0$$
$$x \geq -5$$

Therefore, the domain is: D: $[-5, \infty)$

To determine the range, we note that the square root of a positive number will also be positive, and therefore the range is: $R: [0, \infty)$. See the graph below for additional insight.




$D: [-5, \infty)$

$R: [0, \infty)$

c. $f(x) = \frac{1}{x^2-2x-3}$

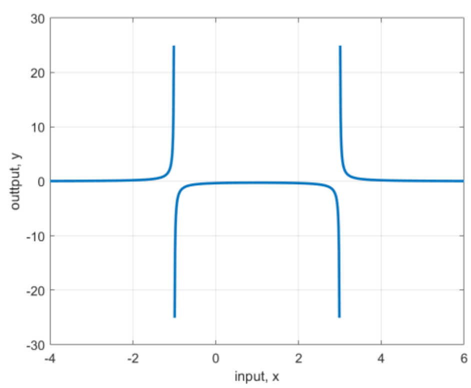
In this case, since we cannot divide by zero, the domain can be found as follows:

$$x^2 - 2x - 3 \neq 0$$
$$(x-3)(x+1) \neq 0$$

$$x \neq 3, x \neq -1$$

Therefore, the domain is: $D: (-\infty, -1) \cup (-1,3) \cup (3, \infty)$.

Notice that $f(x)$ in this example is a rational function for which we learned how to graph in our algebra review. One of the main steps to graphing these functions is to find the vertical and horizontal asymptotes. The vertical asymptotes were found above when solving for the domain. To find the range of the function it may seem that we need only find the horizontal asymptotes and remove these values from the range of the function. However, recall that a rational function *may* or *may not* cross a horizontal asymptote. In this case its best to use a graphing calculator to find the range. The results are shown in the figure below.




$D: (-\infty, -1) \cup (-1,3) \cup (3, \infty)$

$R: (-\infty, 0) \cup (0, \infty)$

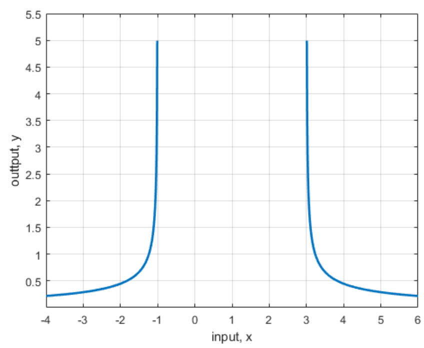
d. $f(x) = \frac{1}{\sqrt{x^2-2x-3}}$

Note this function is like the previous except there is a square root function in the denominator. Therefore, in addition to not allowing a zero in the denominator the function inside the square root must also remain positive. The domain is then found as follows:

$$x^2 - 2x - 3 \geq 0$$

Recall from part $a$ this parabola is concave up and therefore it is negative between the $x$-intercepts. The domain is: $D: (-\infty, -1) \cup (3, \infty)$

We can again use a graphing calculator to find the range, however, because of the square root, the output of this function will always be positive. Therefore, the range is: $R: (0, \infty)$. See the graph below for additional insight.




$D: (-\infty, -1) \cup (3, \infty)$

$R: (0, \infty)$

**Algebra of Functions:**

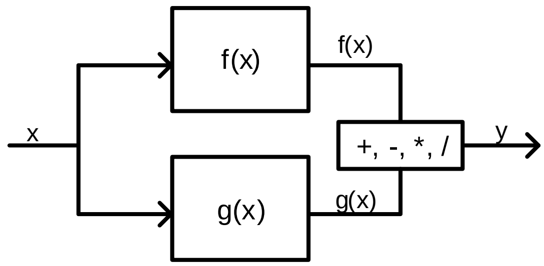
When you evaluate a real valued function at a given $x$ value the result is a real number. For example, evaluating the function, $f(x) = 2x + 3$, at $x = 3$ results in a new value, $y = 9$.

$$f(3) = f(x)|_{x=3} = 2(3) + 3 = 9$$

It seems logical therefore that we can treat a function just like a number and perform various algebraic operations on it. We start by defining the basic arithmetic operations of addition, subtraction, multiplication, and division for two functions, $f(x)$ and $g(x)$, below.

| | |
|---|---|
| **Sum:** $(f+g)(x) = f(x) + g(x)$ | **Product:** $(f \cdot g)(x) = f(x) \cdot g(x)$ |
| **Difference:** $(f-g)(x) = f(x) - g(x)$ | **Quotient:** $(f/g)(x) = f(x)/g(x)$ |

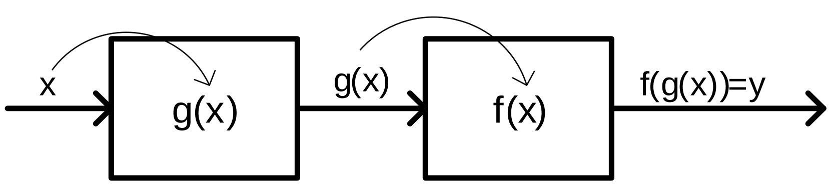
We can also look at these operations in terms of function machines mentioned above.




Let's take the example where $f(x) = 2x + 3$, and $g(x) = 3x^2 + 4x + 6$, and construct four new functions, $h(x)$, as follows:

| **Sum:** | **Product:** |
|---|---|
| $h(x) = (f+g)(x)$<br>$h(x) = f(x) + g(x)$<br>$h(x) = (2x+3) + (3x^2+4x+6)$<br>$h(x) = 3x^2 + 6x + 9$ | $h(x) = (f \cdot g)(x)$<br>$h(x) = f(x) \cdot g(x)$<br>$h(x) = (2x+3)(3x^2+4x+6)$<br>$h(x) = 6x^3 + 8x^2 + 12x + 9x^2 + 12x + 18$<br>$h(x) = 6x^3 + 17x^2 + 24x + 18$ |
| **Difference:**<br>$h(x) = (f-g)(x)$<br>$h(x) = f(x) - g(x)$<br>$h(x) = (2x+3) - (3x^2+4x+6)$<br>$h(x) = -3x^2 - 3x - 3$ | **Quotient:**<br>$h(x) = (f/g)(x)$<br>$h(x) = f(x)/g(x)$<br>$h(x) = \frac{(2x+3)}{(3x^2+4x+6)}$ |

Any practical application of math requires not only the arithmetic of functions, but also a so-called *composition* of functions. When two functions are composed the output of one function is the input to the next. This is most easily described using the idea of function machines as shown below.



As an example, let's take the same two functions from earlier and form the composition depicted in the figure above. Note we can also interchange the ordering of the functions. Furthermore, the formal notation for composition, $(f \circ g)(x)$**,** is also shown.

| $\boldsymbol{h(x) = (f \circ g)(x)}$ | $\boldsymbol{h(x) = (g \circ f)(x)}$ |
|---|---|
| $h(x) = f\big(g(x)\big)$ | $h(x) = g\big(f(x)\big)$ |
| $h(x) = f(3x^2 + 4x + 6)$ | $h(x) = g(2x + 3)$ |
| $h(x) = 2(3x^2 + 4x + 6) + 3$ | $h(x) = 3(2x + 3)^2 + 4(2x + 3) + 6$ |
| $h(x) = 6x^2 + 8x + 12 + 3$ | $h(x) = 3(4x^2 + 12x + 9) + 8x + 12 + 6$ |
| $h(x) = 6x^2 + 8x + 15$ | $h(x) = 12x^2 + 44x + 45$ |

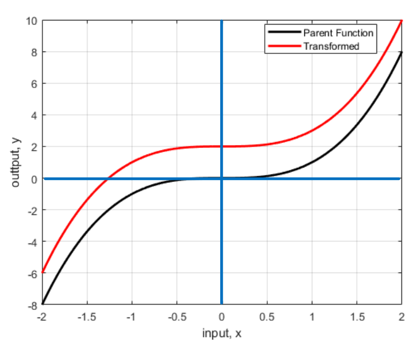
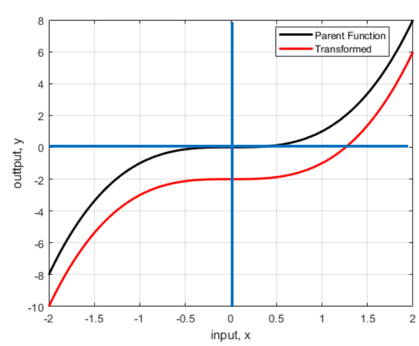
**Function Transformations**

Our final review topic is on function transformations. As you may recall, the three basic function transformations involve: 1.) shifting a function up, down, left, or right 2.) reflecting a function across the $x$ or $y$ axis. 3.) vertically or horizontally scaling a function. You should also recall that when working with transformation we generally try to first identify the so-called *parent function*. As a review below we define the six fundamental transformations and provide an example for each using the parent function, $p(x) = x^3$.

*1. Vertical Shift*

| | |
|---|---|
| Shift Up | $f(x) = p(x) + A$ |
| Shift Down | $f(x) = p(x) - A$ |

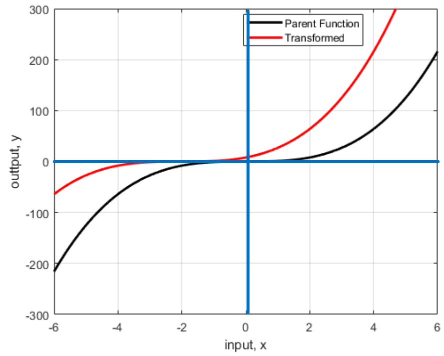
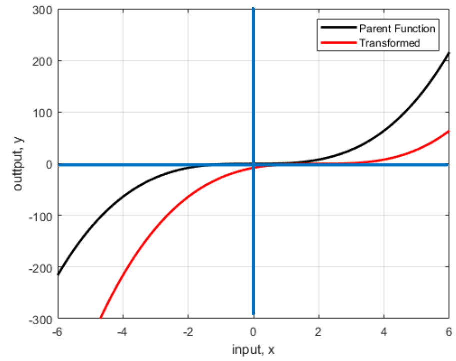
The figures below show both cases with $A = 2$.

| $f(x) = x^3 + 2$ | $f(x) = x^3 - 2$ |
|---|---|
|   |   |

## 2. *Horizontal Shift*

| | |
|---|---|
| Shift Left | $f(x) = p(x + A)$ |
| Shift Right | $f(x) = p(x - A)$ |

The figures below show both cases with $A = 2$.

| $f(x) = (x + 2)^3$ | $f(x) = (x - 2)^3$ |
|---|---|
|  |  |

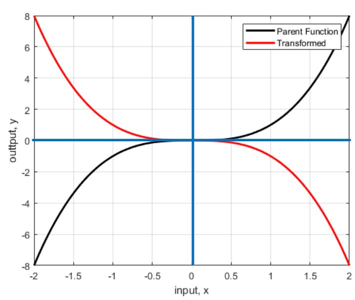
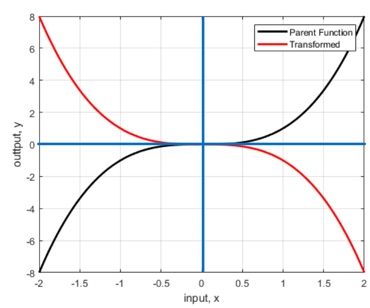
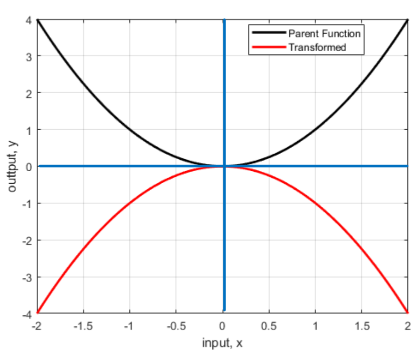
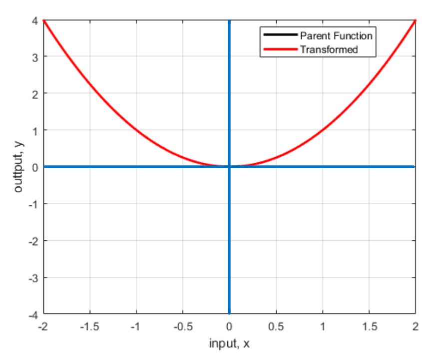


## 3. *X-axis Reflection*

## 4. *Y-axis Reflection*

| | |
|---|---|
| $x$-axis reflection | $f(x) = -p(x)$ |
| $y$-axis reflection | $f(x) = p(-x)$ |

The figures below show both cases. Note that for $p(x) = x^3$ the plots are identical because when cubing a negative number, it remains negative. To highlight the different reflections, we also show the case with $p(x) = x^2$.

| $f(x) = -(x^3) = -x^3$ | $f(x) = (-x)^3 = -x^3$ |
|---|---|
|  |  |
| $f(x) = -(x^2) = -x^2$ | $f(x) = (-x)^2 = x^2$ |
|  |  |

*5. Vertical Stretching or Shrinking*

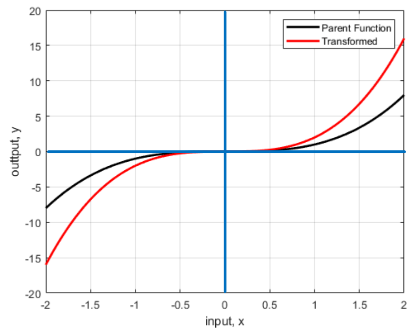
$$f(x) = Ap(x)$$

Where we have:

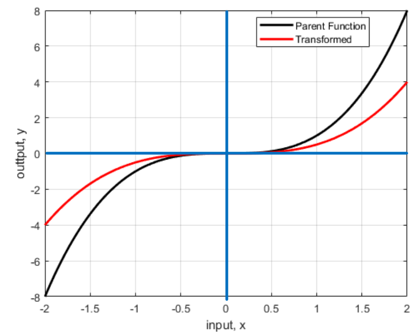
| | |
|---|---|
| Stretching | $\lvert A \rvert > 1$ |
| Shrinking | $0 < \lvert A \rvert < 1$ |

The figures below show both cases with $A = 2$ and $A = \frac{1}{2}$.

$f(x) = 2x^3$




$f(x) = (1/2)x^3$




*6. Horizontal Stretching or Shrinking*

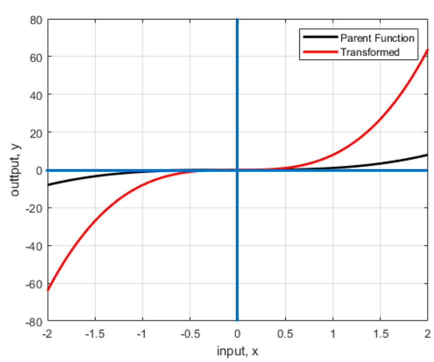
$$f(x) = p(Ax)$$

Where we have:

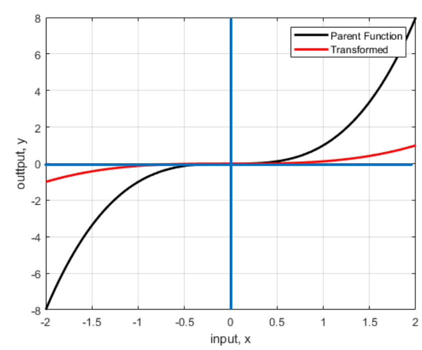
| | |
|---|---|
| Shrinking | $\lvert A \rvert > 1$ |
| Stretching | $0 < \lvert A \rvert < 1$ |

The figures below show both cases with $A = 2$ and $A = \frac{1}{2}$. Note that in this case horizontal stretching/shrinking can also be look at as vertical shrinking/stretching with modified scale factor.

$f(x) = (2x)^3 = 8x^3$




$f(x) = \left(\frac{1}{2}x\right)^3 = \frac{1}{8}x^3$



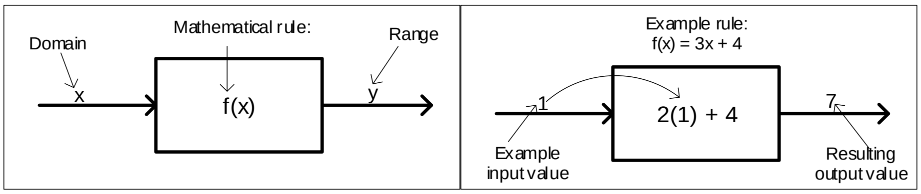
# ***Final Summary for Pre-Calc Functions Review***

## Function Definition

A ***function*** is a mathematical rule that assigns a *unique* output value to every input value.

- The set of allowable input values, usually denoted by $x$, is called the *domain* of the function.
- The set of all output values, usually denoted by $y$, is called the *range* of the function.
- The function itself is usually denoted by $f$.
- Applying an input value, $x$, to s function $f$, to produce an output value is denoted as:

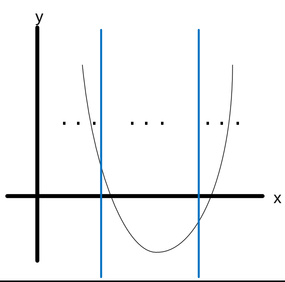
$$y = f(x)$$



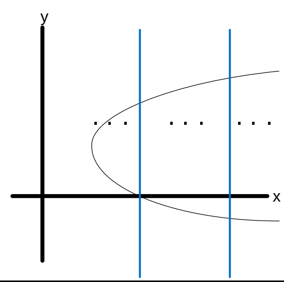

## Vertical Line Test

*Vertical Line Test:* If a vertical line passes through a curve more than once the relationship is not a function.

**Function**




**Not a Function**




## Linear Function

A linear function is one in which the graph is a straight line. Algebraically, linear functions are usually expressed in one of three different forms shown below.

| | | |
|---|---|---|
| **Slope Intercept Form** | $y = mx + b$ | $m$= slope, $b$ = y-intercept |
| **Point Slope Form** | $(y - y_1) = m(x - x_1)$ | $m$= slope, $(x_1, y_1)$ is a point on the line. |
| **Standard Form** | $Ax + By = C$ | $A, B$, and $C$ are constants |

Where,

$$m = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

- *Parallel lines* have the same slope, i.e. $m_2 = m_1$
- *Perpendicular lines* have slopes that are the negative inverses of each other, $m_2 = -\frac{1}{m_1}$

## Quadratic Function

A quadratic function is a polynomial of degree 2, which is normally represented in different forms as shown below:

| | | |
|---|---|---|
| **Standard From** | $f(x) = ax^2 + bx + c$ | $a, b,$ and $c$ are constants |
| **Vertex Form** | $f(x) = a(x-h)^2 + k$ | the point $(h, k)$ is the vertex. |

In standard form the vertex is given as:

$$(h,k) = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

The *quadratic formula* can be used to find the roots, (where $f(x) = 0$).

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The *discriminate,* $D$*,* can be used to describe the types of roots.

$$D = b^2 - 4ac$$

| $\boldsymbol{D > 0}$<br>2 real roots | $\boldsymbol{D = 0}$<br>1 repeated root | $\boldsymbol{D < 0}$<br>2 complex conjugate roots |
|---|---|---|
| $x_{1,2} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | $x = -\frac{b}{2a}$ | $x_{1,2} \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$ |

## Piecewise Function

A piecewise function is one that is defined differently for different ranges of inputs. A general definition can be given as follows:

$$f(x) = \begin{cases} f_1(x), & x < a_1 \\ f_2(x), & a_1 \le x < a_2 \\ f_3(x), & a_2 \le x < a_3 \\ & \quad . \\ & \quad . \\ & \quad . \\ f_n(x), & a_{n-1} \le x < a_n \end{cases}$$

**Domain and Range of Functions**

**Domain of a function, $f$:**

The set of all possible $x$ values that when applied to the function produce a *valid* output value, $y$.

Note: For real valued functions a valid output is any real number.

**Range of a function, $f$:**

The set of all values, *y*, that are produced by the function as a result of applying all values, *x*, in the domain.

**Algebra of Functions**

**Arithmetic**

**Sum:**

$$(f + g)(x) = f(x) + g(x)$$

**Product:**

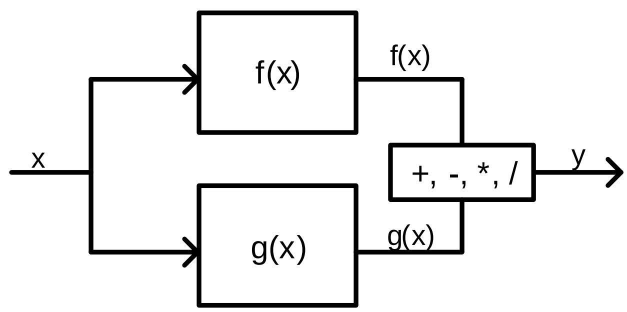
$$(f \cdot g)(x) = f(x) \cdot g(x)$$

**Difference:**

$$(f - g)(x) = f(x) - g(x)$$

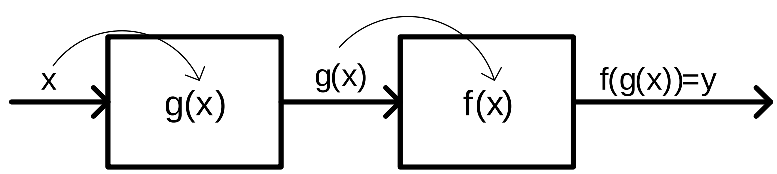
**Quotient:**

$$(f/g)(x) = f(x)/g(x)$$




**Composition**

$$(f \circ g)(x) = f\big(g(x)\big)$$



| Function Transformations |
| --- |
| There are six fundamental function transformations. Transforming the parent function, $p(x)$, to a new function, f$(x)$.<br><br>1. *Vertical Shift*<br>• *Shift Up:* $f(x) = p(x) + A$<br>• *Shift Down:* $f(x) = p(x) - A$<br>2. *Horizontal Shift*<br>• *Shift Right:* $f(x) = p(x - A)$<br>• *Shift Left:* $f(x) = p(x + A)$<br>3. *X-Axis Reflection*<br>• $f(x) = -p(x)$<br>4. *Y-Axis Reflection*<br>• $f(x) = p(-x)$<br>5. *Vertical Stretching/Shrinking*<br>• $f(x) = Ap(x)$<br>i. *Stretching:* $\lvert A \rvert > 1$<br>ii. *Shrinking:* $0 < \lvert A \rvert < 1$<br>6. *Horizontal Stretching/Shrinking*<br>• $f(x) = p(Ax)$<br>i. *Stretching:* $0 < \lvert A \rvert < 1$<br>ii. *Shrinking:* $\lvert A \rvert > 1$ |

*By:* ferrantetutoring