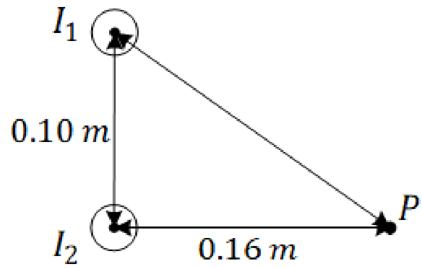
Magnetic Field from Currents Examples

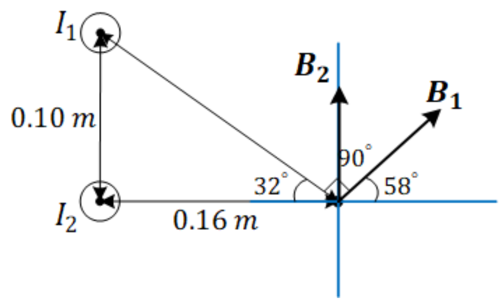
Question 1:

Two long thin parallel wires shown in the figure below carry a current of $25\ A$ in the same direction. Find the direction and magnitude of the magnetic field at point P.



Solution 1:

The magnetic field at point P can be found by taking the vector sum of the magnetic field from each wire. We start by setting up a coordinate system at the point P. The magnetic field vector from each wire is perpendicular to the line drawn from the wire to the point P.



The direction of the magnetic field from I_2 is directed vertically up.

The direction of the magnetic field from I_1 is determined as follows:

The angle from the point P to I_1 is given as:

$$\tan^{-1}\left(\frac{0.10}{0.16}\right) = 32^\circ$$

$\boldsymbol{B_1}$ is drawn 90° from a line drawn from the point P to the wire carrying I_1. With this we can find the angle from the positive x axis to $\boldsymbol{B_1}$ as

$$\theta = 180^\circ - (90^\circ + 32^\circ) = 58^\circ$$

The magnitude of $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ is found as follows:

$$B_{1/2} = \frac{\mu_0 I}{2\pi d_{1/2}}$$

Where, d_1 is the hypotenuse of the right triangle

$$d_1 = \sqrt{0.10^2 + 0.16^2}$$
$$d_1 = 0.19\ m$$

Therefore, the magnitude of $\boldsymbol{B_1}$ is

$$B_1 = \frac{4\pi E^{-7} \cdot 25}{2\pi \cdot 0.19}$$
$$B_1 = 2.63\ E^{-5}$$

And since d_2 is the length of the base of the triangle, B_2 is

$$B_2 = \frac{4\pi E^{-7} \cdot 25}{2\pi \cdot 0.16}$$
$$B_2 = 3.13\ E^{-5}$$

To find the total $\boldsymbol{B}$ field we add the vectors $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ as follows:

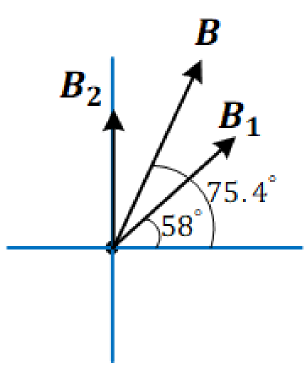
$$\boldsymbol{B} = \boldsymbol{B_1} + \boldsymbol{B_2}$$
$$\boldsymbol{B} = 2.63\ E^{-5}\langle \cos(58), \sin(58) \rangle + 3.13\ E^{-5}\langle \cos(90), \sin(90) \rangle$$
$$\boldsymbol{B} = 2.63\ E^{-5}\langle 0.53, 0.85 \rangle + 3.13\ E^{-5}\langle 0, 1 \rangle$$
$$\boldsymbol{B} = \langle 1.4\ E^{-5}, 5.37\ E^{-5} \rangle$$

The magnitude and angle from the x-axis are then found as:

$$B = \sqrt{(1.4\ E^{-5})^2 + (5.37\ E^{-5})^2}$$
$$B = 5.55\ E^{-5}\ T$$

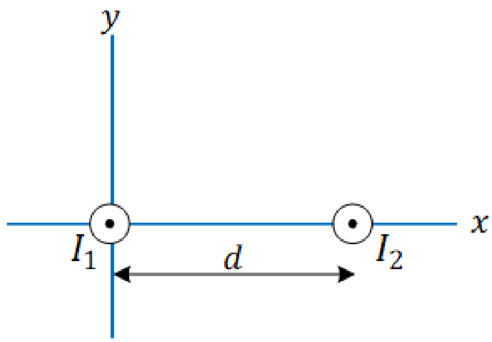
$$\theta_B = \tan^{-1}\left(\frac{5.37}{1.4}\right)$$
$$\theta_B = 75.4^{\circ}$$

And illustrated below for clarity.



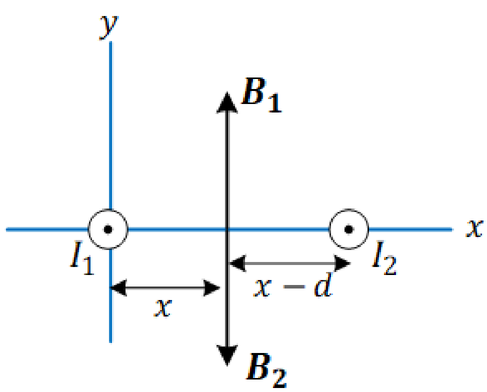
Question 2:

There are two parallel wires a distant $d = 1\ m$ apart carrying the same current of $50\ A$ as shown below. Determine $\boldsymbol{B}$ between the wires as a function of x



Solution 2:

Between the wires the magnetic field is in the positive y direction from the first wire and in the negative y direction from the second wire.



The resulting field is then given as:

$$B = B_1 - B_2$$

$$B = \frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(d-x)}$$

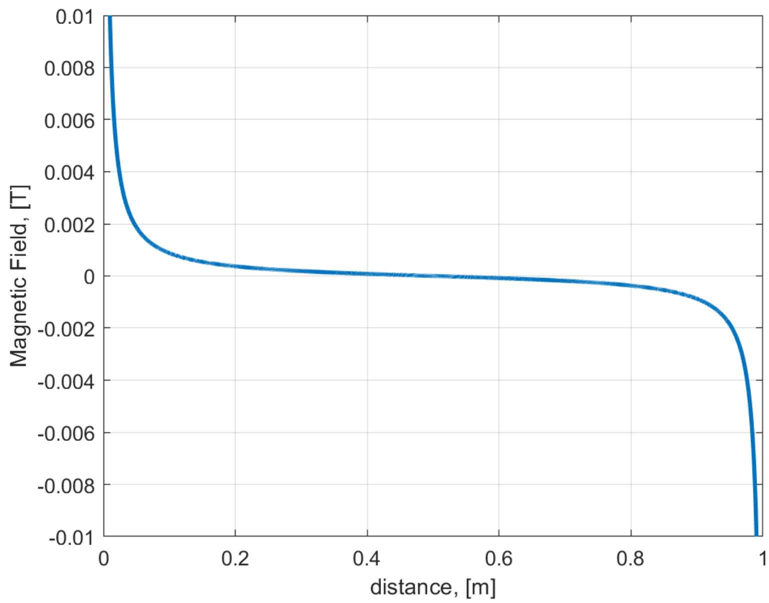
$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{(d-x)}\right)$$

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{(d-x)-x}{x(d-x)}\right)$$

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{d-2x}{x(d-x)}\right)$$

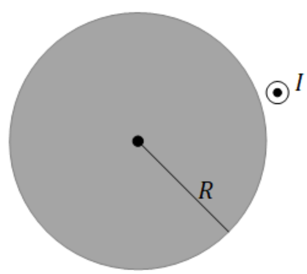
$$B = 1\ E^{-5}\left(\frac{1-2x}{x(1-x)}\right)$$

To gain more insight we plot the B as a function of x below.



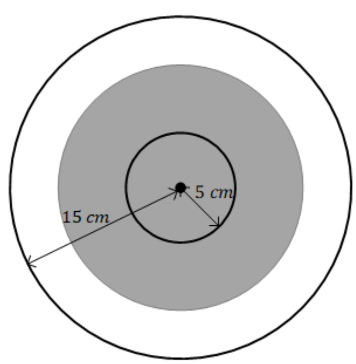
Question 3:

A cylindrical cable is carrying a current of $25\ A$. The cable has a radius, R, of $10\ cm$. Find the magnetic field inside the cable $5\ cm$ from the center, and outside the cable $15\ cm$ from the center.



Solution 3:

Since the magnetic field has cylindrical symmetry we use circles for our Amperian loops.



Ampere's law is then written as follows:

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 I_{enc}$$
$$B \int_0^{2\pi r} dl = \mu_0 I_{enc}$$
$$B 2\pi r = \mu_0 I_{enc}$$
$$B = \frac{\mu_0 I_{enc}}{2\pi r}$$

Which gives us the magnitude of the magnetic field as a function of the distance from the cable center, r. When $r = 5\ cm$ not all the current is enclosed inside our Amperian loop. However, if we assume the current is uniform we can use the following ratio to determine the amount of current inside the loop.

$$\frac{I_{enc}}{I_T} = \frac{\cancel{2\pi} r}{\cancel{2\pi} R}$$
$$I_{enc} = I_T \frac{r}{R}$$
$$I_{enc} = 25 \cdot \frac{5}{10}$$
$$I_{enc} = 12.5\ A$$

Furthermore, when $r = 15\ cm$ we have $I_{enc} = I_T = 25\ A$.

Knowing these values, we solve for the magnetic field inside and outside the cable below.

Inside Cable @ $r = 5\ cm$

$$B = \frac{\mu_0 I_{enc}}{2\pi r}$$
$$B = \frac{4\pi\ E^{-7} \cdot 12.5}{2\pi \cdot 0.05}$$
$$B = 5\ E^{-5} T$$

Outside Cable @ $r = 15\ cm$

$$B = \frac{\mu_0 I_{enc}}{2\pi r}$$
$$B = \frac{4\pi\ E^{-7} \cdot 25}{2\pi \cdot 0.15}$$
$$B = 3.3\ E^{-5}\ T$$

Question 4:

A solenoid with a radius of $r_s = 1.5\ cm$ is made from a $20\ m$ long copper wire. The wire has a diameter of $2.0\ mm$ including insulation. Find the length of the solenoid and the magnetic field at the center of the solenoid if it carries a current of $20\ A$.

Solution 4:

Each loop of wire requires a length of $2\pi r_s = 2\pi \cdot 0.015 = 0.094\ m$. The number of loops that can be created using the 20 m is then

$$N = \left\lfloor \frac{20}{0.094} \right\rfloor$$

$$N = 212$$

Where we used the $\lfloor x \rfloor$ operator, which indicates the greatest integer less that x, so that we have a whole number of loops.

The length of the solenoid is determined by the diameter of the wire. Assuming the loops of wire are tightly wound together the length of the solenoid is

$$L = 212 \cdot 0.002$$

$$L = 0.424\ m$$

Finally, the magnetic field inside the solenoid is given as:

$$B = \mu_0 n I$$

$$B = \mu_0 \frac{N}{L} I$$

$$B = 4\pi\ E^{-7} \cdot \frac{212}{0.424} \cdot 20$$

$$B = 1.26\ E^{-2}\ T$$

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