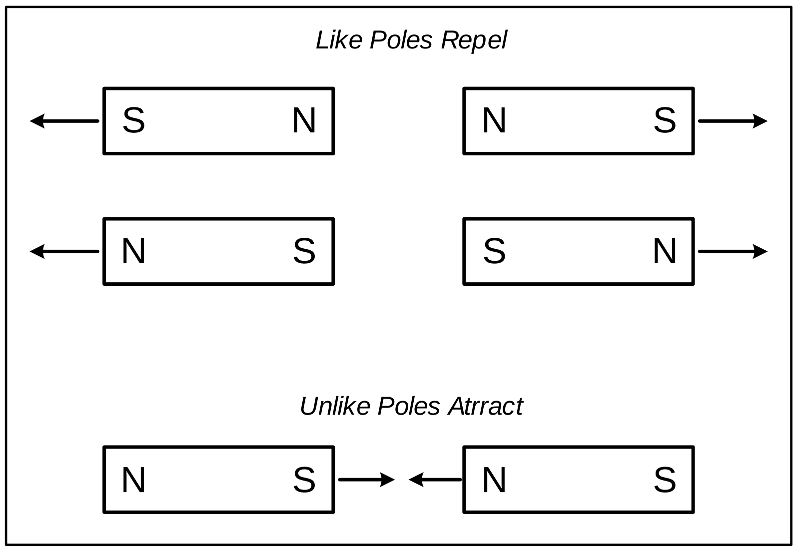
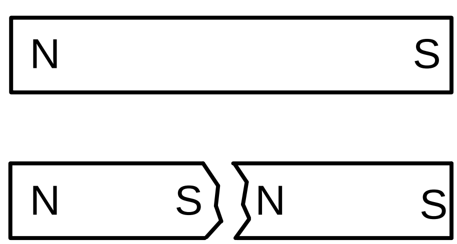
Magnetism Introduction

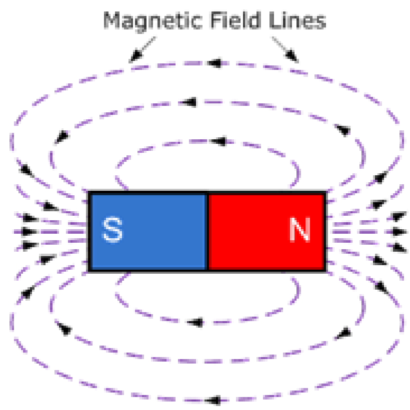
Most of us are familiar with the common "refrigerator magnet", and the fact that magnets attract, (i.e. exert a force on), objects made if iron. Materials that show strong magnetic effects are referred to as ferromagnetic. They include mainly iron, nickel, and cobalt. We are likely also familiar with the fact that magnets have a north and south pole that follow similar attractive and repulsive laws that electric charges do, mainly; like poles attract and unlike poles repel.



An important difference however is that positive and negative electric charges can exists in isolation, whereas isolated magnetic poles have not so far been found to exist. For example, if we break a magnet in half we *do not* create isolated north and south poles. Instead two new magnets are produced as shown below.

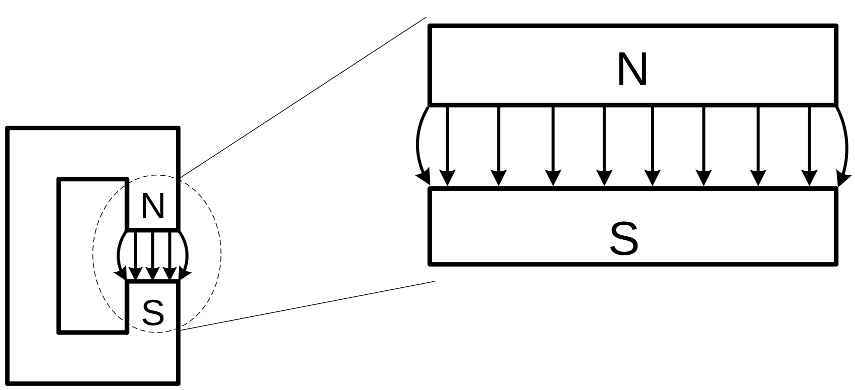


We would like to study the forces that are produced from magnets (i.e. the magnetic force). Recall when studying the electric force, we found it extremely useful to utilize the concept of the electric field. We can similarly imagine a magnetic field that surrounds magnets. However, since magnetic poles do not exist in isolation, magnetic field lines always form a closed loop between the north and south poles. The magnetic field lines surrounding a bar magnet are shown below for illustration. The lines are shown emerging from the north pole and terminating at the south pole, however, the lines do indeed pass through the magnet and form closed loops.

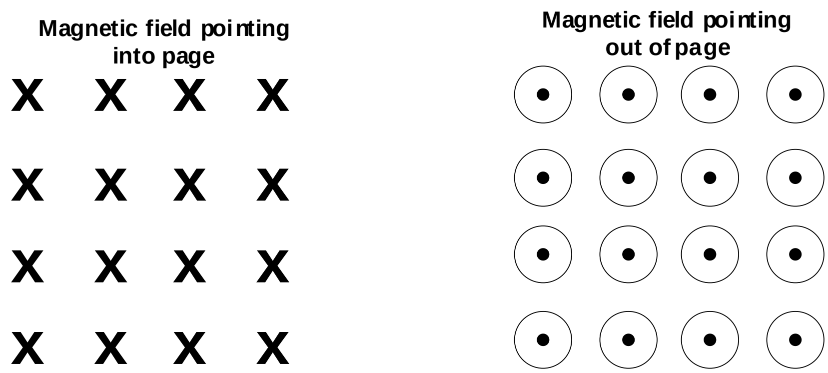


The effects of the magnetic force on iron and other magnets is quite an interesting phenomenon and has been known for thousands of years. More interesting, however, was the discovery of the intimate relationship between electricity and magnetism, which wasn't recognized until the nineteenth century. Discovering this detailed relationship took much time and effort and culminated with Maxwell's mathematical description of the electromagnetic field, but that's getting ahead of ourselves.

Before we begin let's note two concepts that will be used throughout our study of magnetism. First is that in many of our examples we will refer to a "uniform magnetic field", in which the field lines are parallel. One way this can be created is by bending a magnet into a C-shape as shown below. As you can see the field can be considered nearly uniform except at the edges.



The next concept has to do with the fact, (which will soon become apparent), that we need to consider three-dimensional space in most of our analysis. To avoid always needing to draw three dimensional pictures our third dimension will sometimes be considered perpendicular to the plane of the paper. With this, for a magnetic field that is pointing into the page we draw x's, which are meant to resemble the tail of an arrow moving away from the reader, and for a field that is pointing out of the page we draw dots, which are meant to represent the tip of an arrow pointing towards the reader.



Magnetic Effects on Electric Charges

We begin exploring the interaction of electricity and magnetism by first placing a positively charged particle in a uniform magnetic field. When we do this, we realize that the charge is unaffected. However, if we put this charge in motion we observe that the path of the charge *is* affected by the magnetic field! To find the precise affects we perform many trials, each time giving the charge a different speed and direction with respect to the magnetic field. If we do this, we find a vector-based relationship that uses the notion of a vector cross product.

$$\boldsymbol{F}_B = q\boldsymbol{v}\, x\, \boldsymbol{B}$$

Where the magnetic field vector, $\boldsymbol{B}$, is given the unit of a tesla (T), $\boldsymbol{v}$ is the velocity of the charge in units of meters per second, and q is the charge of the particle in Coulombs. Finally, the force on the particle due the magnetic field is $\boldsymbol{F}_B$, measured in Newtons. To make sense of this equation let's first review the vector cross product.

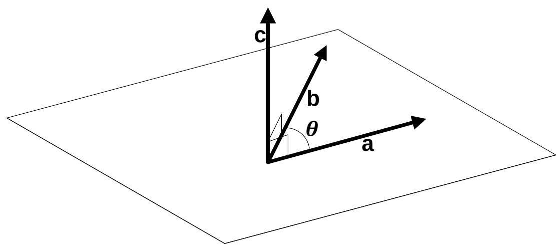
Vector Cross Product:

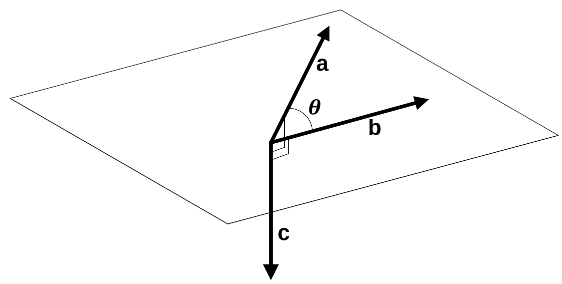
Direction:

Given two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, the vector cross product, $\boldsymbol{a}\, x\, \boldsymbol{b}$, results in a new vector, $\boldsymbol{c}$, that is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$. We write the vector cross product as follows:

$$\boldsymbol{c} = \boldsymbol{a}\, x\, \boldsymbol{b}$$

Since the vector $\boldsymbol{c}$ is perpendicular to the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, it will either be pointing towards or away from the plane formed by the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. These two options are determined based on what is referred to as the "right-hand rule", which is illustrated in the figures below. To perform the right-hand rule, we use our right hand and place our fingers in the direction of the first vector, $\boldsymbol{a}$, with our palm facing the second vector, $\boldsymbol{b}$. When we close our hand by sweeping our fingers from $\boldsymbol{a}$ to $\boldsymbol{b}$, the thumb will point in the direction of the vector $\boldsymbol{c}$.





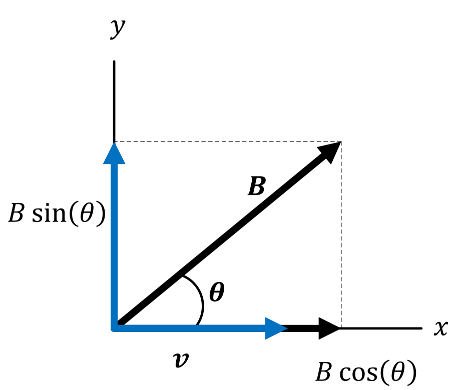
Magnitude:

Mathematically the magnitude of the vector cross product is given as follows:

$$c = ab\sin(\theta)$$

Where, θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

However, there is an alternative way to interpret this equation that is more instructive from a physics perspective. Let's use the vectors $\boldsymbol{v}$ and $\boldsymbol{B}$ to make the discussion less abstract. The figure below focuses on the 2D plane formed by $\boldsymbol{v}$ and $\boldsymbol{B}$ since we have already determined the direction of the resulting vector above. We show the velocity vector pointing in the positive x direction and the magnetic field vector pointing at an angle, θ, measured from the positive x-axis.



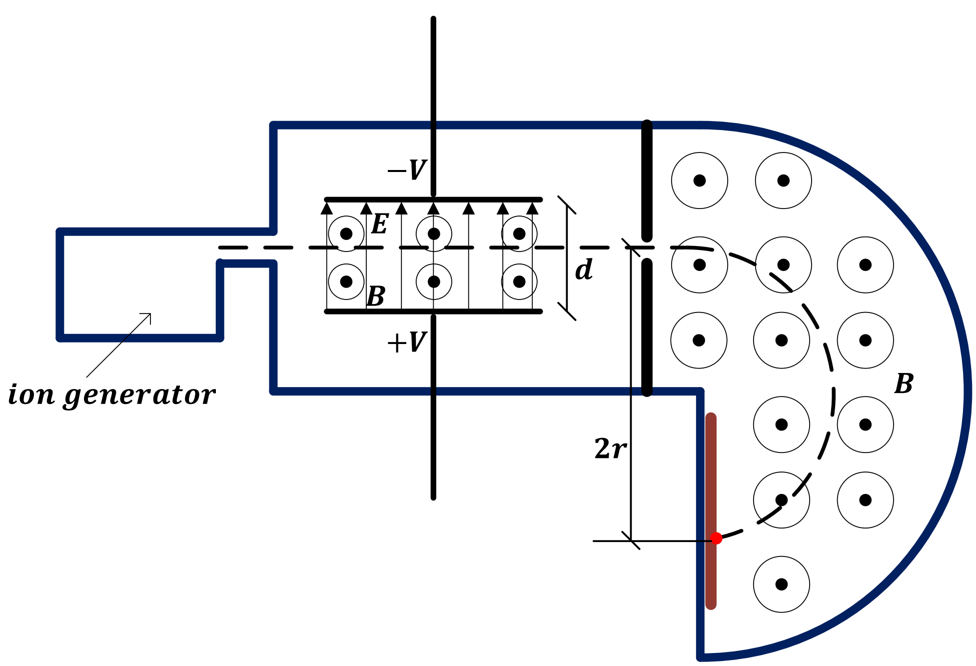
The figure also shows the field vector, $\boldsymbol{B}$, decomposed into its x and y components. The cross-product relationship accounts for the fact that only the field component that is *perpendicular* to the velocity vector contributes to the force on the particle. The parallel component, $B\cos(\theta)$, does not contribute to the force on the charge. Multiplying the magnitude of the velocity vector with the perpendicular component of the magnetic field vector gives us the same equation as above, but with a much more intuitive understanding.

$$F = v \cdot (B\sin(\theta))$$

With the vector cross product understood, the original equation, shown again below, should make a bit more sense now.

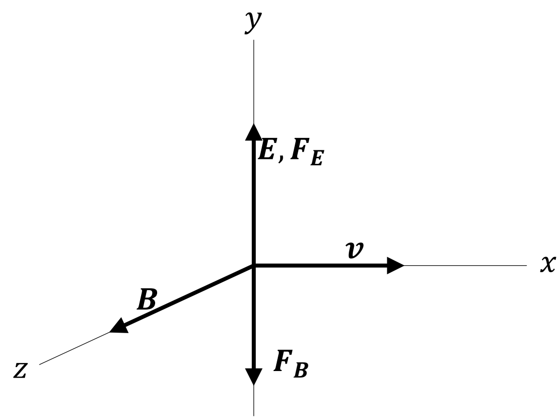
$$\boldsymbol{F}_B = q\boldsymbol{v}\ x\ \boldsymbol{B}$$

An interesting application of this newfound relationship is a method that was developed in the early 1900's to measure the masses of atoms. The technique is called mass spectrometry, and the device used is called a mass spectrometer. The figure below shows a simplified version of a mass spectrometer. As an example, positively charged ions are produced and enter the device from the "ion generator" shown on the left. We assume the ions produced are of different type, (i.e. masses), but that they are all charged to the same positive value due to a single missing electron. The ions first pass through a region of crossed electric and magnetic field, which as we will show, acts as a velocity selector. The "selected" ions then enter a second region with another uniform magnetic field where the ions straight-line path will begin to curve, due to the magnetic field, into a circular path with a mass dependent radius. Finally, the ions will hit a photographic plate so that the location of impact can be determined. Let's now examine the regions in more detail.



Region 1- Velocity Selector:

The ions leave the "ion generator" with various velocities and enter the region where the electric and magnetic field are oriented as shown below.



Based on the above figure we see that the electric force will act to push the particle in the positive y direction, while the magnetic field acts to push the particle in the negative y direction. Equating the magnitude of these two forces allows us to find the velocity of the ions that experience no net force and therefore continue a straight-line path to enter the second region.

$$F_B = F_E$$

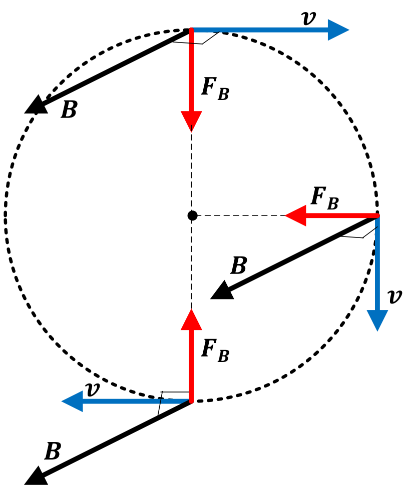
$$\cancel{q}v(B\sin(90)) = \cancel{q}E$$

$$v = \frac{E}{B}$$

Note that by adjusting the strength of the electric and/or magnetic field we can "select" ions that are traveling at a specific velocity to enter the second region.

Region 2- Mass Determination:

The selected ions now enter the second region with the magnetic field aligned again in the positive z-direction. Note since the magnetic force is always perpendicular to both the velocity and magnetic field vector, the ions will be forced to change in *direction only* and therefore experience uniform circular motion as shown below.



Recall for uniform circular motion the radial force is given as follows:

$$F_r = \frac{mv^2}{r}$$

Since the magnetic force is responsible for providing this radial force we can equate the two forces and solve for the mass of the ion!

$$F_r = F_B$$
$$\frac{mv^{\not{2}}}{r} = q\not{v}B$$
$$m = \frac{qBr}{v}$$

Let's work through the equations using the following values. Let the distance between the plates in the first region, d, be $0.01\ m$ and the applied a voltage be $1700\ V$. The electric field strength can then be found as below.

$$E = \frac{\Delta V}{\Delta d}$$
$$E = \frac{1700}{0.01}$$
$$E = 1.7\,E^5\ V/m$$

Furthermore, using a value of $B = 0.3\ T$ the velocity of the ions entering the second region can also be found.

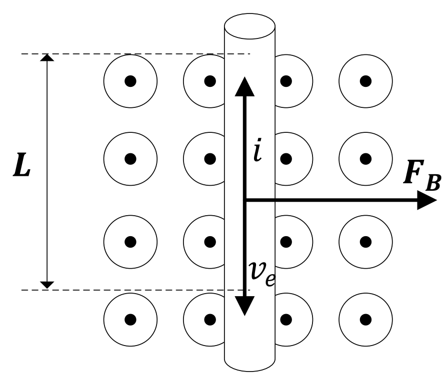
$$v = \frac{E}{B}$$
$$v = \frac{1.7\ E^5}{0.03}$$
$$v = 5.7\ E^5\ m/s$$

Finally, if the radius is measured to be $0.15\ m$, the mass of the ion can be found.

$$m = \frac{qBr}{v}$$
$$m = \frac{1.6\ E^{-19} \cdot 0.03 \cdot 0.15}{5.7\ E^5}$$
$$m = 1.26\ E^{-27}\ kg$$

Magnetic Force on a Current Carrying Wire

We now know that charged particles moving through a magnetic field may experience a magnetic force. One common place we find moving charges, (i.e. electrons), are in current carrying wires. Since the electrons are confined inside the wire they will in turn exert a force on the wire itself. Let's see if we could find an expression for the magnetic force that is exerted on a current carrying wire placed in a uniform magnetic field.



The figure above shows a current carrying wire placed in a magnetic field. Recall that it's the negatively charged electrons that are traveling in the wire and they are traveling in the direction of $\boldsymbol{v_e}$. Using the cross product from our magnetic force equation, $\boldsymbol{v_e}\ x\ \boldsymbol{B}$, will result in a magnetic force vector pointing to the left. However, to compute the final force vector we multiply by the charge, which in this case is negative, so that the magnetic force vector ends up pointing to the right as shown. We should note that the same result is obtained by considering positively charged particles moving in the direction of the current. Now that we have established the direction of the force, let's analyze the magnitude while trying to relate the force to the current in the wire. As the current is defined as the rate of change of the charge, we can find the charge by multiplying this expression by dt and integrate over a time, t, as show below.

$$i = \frac{dq}{dt}$$
$$\int_0^q 1\, dq = \int_0^t i\, d\tau$$

And if we consider the current as constant we have:

$$q = it$$

Next, we can replace the time variable by considering a finite length of wire, L, so that we can write $t = \frac{L}{v}$, which upon substitution gives us the following expression for the charge.

$$q = \frac{iL}{v}$$

Finally, substituting this expression for the charge in the magnetic fore equation we can express the magnitude of the magnetic force on a current carrying wire as below.

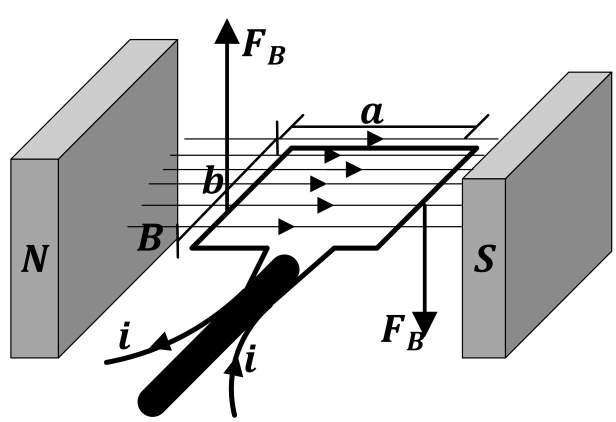
$$F_B = \frac{iL}{\cancel{v}} \cancel{v} B \sin(\theta)$$
$$F_B = iLB \sin(\theta)$$

Combining this result with the direction of the force we determined from above we can write the final equation for the magnetic force vector on a current carrying wire.

$$\boldsymbol{F}_B = i\boldsymbol{L}\, x\, \boldsymbol{B}$$

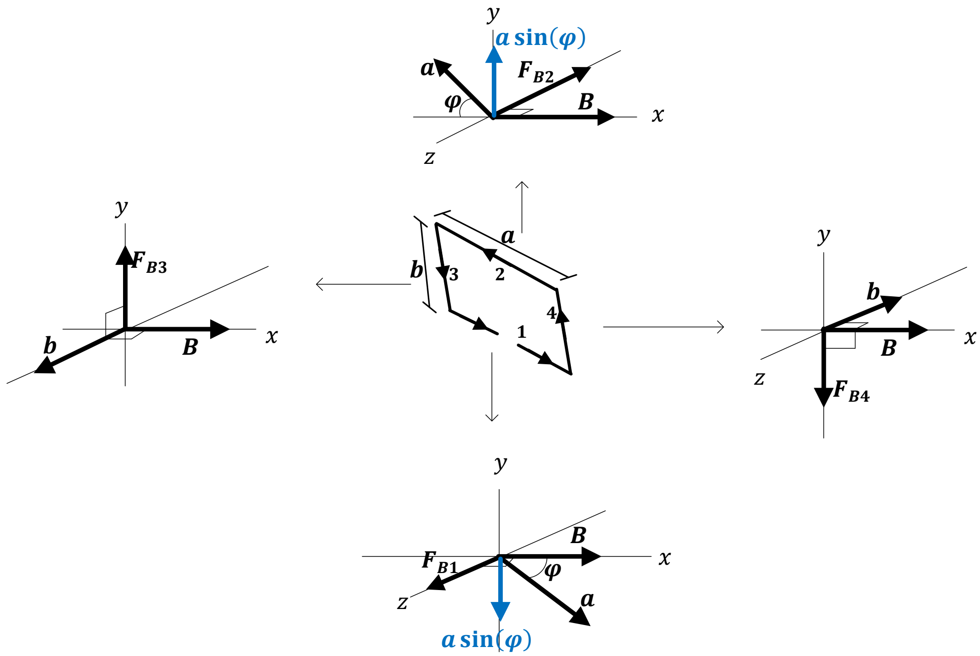
Where the magnitude of $\boldsymbol{L}$ is the length of the wire and points in the direction of the current.

This relationship is fundamental to the operation of the electric motor, which as you may know, is widely used throughout all of society. We examine the basic operation principle below.



The figure above shows a simplified version of an electric motor consisting of a single rectangular rigid loop of current carrying wire placed in a uniform magnetic field.

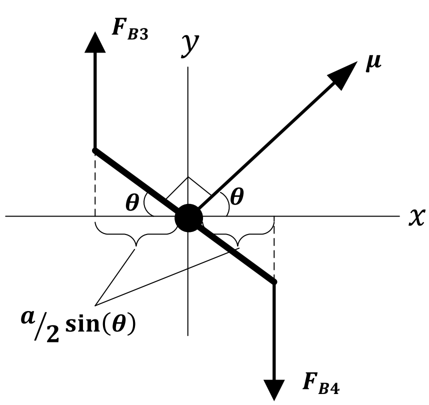
To start let's examine the magnetic force on each side of the rectangle using the figures below.



The table below shows the force acting on each side.

Side 1:	$F_{B1} = ia\sin(\varphi)\,B$	$-z$ *direction*
Side 2:	$F_{B2} = ia\sin(\varphi)\,B$	$+z$ *direction*
Side 3:	$F_{B3} = ibB$	$+y$ *direction*
Side 4:	$F_{B1} = ibB$	$-y$ *direction*

Side 1 and 2 have equal and opposite forces as do sides 3 and 4. However, the forces on side 1 and 2 act along the axis of rotation and therefore produce no torque. On the other hand, the forces on side 3 and 4 act away from and on opposite sides of the axis of rotation and therefore will produce a net torque which will tend to rotate the loop clockwise. Below is an x-y plane view.



Recall from our earlier studies that the torque can be determined by multiplying the force with the lever arm, which in both cases is shown to be ${}^{a}/_{2}\sin(\theta)$, where the angle is shown measured from a vector, $\boldsymbol{\mu}$, (which we will describe below), that is normal to the plane of the loop. The magnitude of the net torque is

$$\tau = F_{B3}\frac{a}{2}\sin(\theta) + F_{B4}\frac{a}{2}\sin(\theta)$$
$$\tau = ibB\frac{a}{2}\sin(\theta) + ibB\frac{a}{2}\sin(\theta)$$
$$\tau = iabB\sin(\theta)$$

Note that the quantity ab is equal to the area, A, of the loop, therefore we can write:

$$\tau = iAB\sin(\theta)$$

Furthermore, to increase the amount of torque we can use N loops of wire to create a "coil". The torque on N wires then becomes

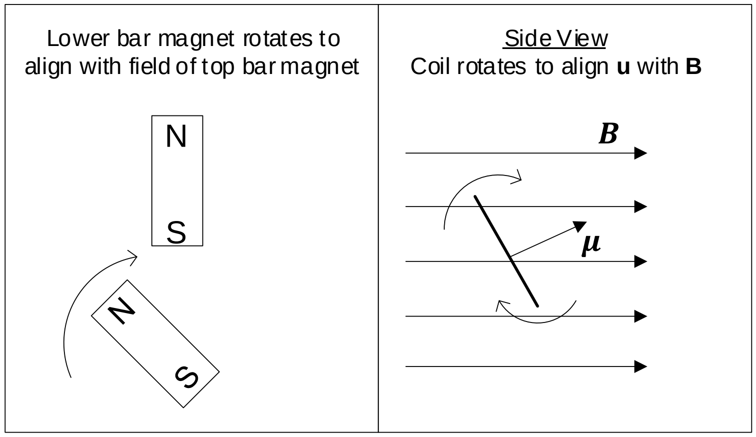
$$\tau = NiAB\sin(\theta)$$

The quantity NiA is called the magnetic dipole moment of the coil and is considered a vector with a direction that is perpendicular to the plane of coil and consistent with the right-hand rule (wrap your fingers in the direction of the current and your thumb points in the direction of the vector). We can write the final vector equation for the torque as follows:

$$\boldsymbol{\tau} = \boldsymbol{u}\, x\, \boldsymbol{B}$$

Where, $\boldsymbol{u} = Ni\boldsymbol{A}$, which was originally shown in the figure above.

With this equation we can solve for the torque on a coil of current carrying wire without needing to analyze each length of wire independently as we did above. Furthermore, to further explain the magnetic dipole vector we first note that the coil behaves as a bar magnet placed in a magnetic field. Therefore, it can be considered a magnetic dipole just as a bar magnet inherently is, and just as a bar magnet would tend to align itself with the direction of the magnetic field, so does the coil. The two cases are shown below for illustration. In the case of the bar magnet we see that the lower magnet would tend to rotate to align the north and south poles. In the case of the coil a torque is established that tends to rotate the coil so that the magnetic dipole moment vector aligns with the direction of the magnetic field. As mentioned, this knowledge can be used to more easily determine the direction of the torque on a coil in a magnetic field.



One last point about the rotation. Once a current begins to flow the loop will begin to rotate clockwise as mentioned. Note, however that when the loop rotates beyond 90° the dipole vector will point below the x-axis. And since the tendency is to align the dipole vector with the field vector, the coil will begin to reverse direction and rotate counterclockwise. If we again pass the 90° point, we get another reversal of the rotation direction. You can imagine that this action would eventually stop the rotation all together. One way to achieve constant rotation would be to switch the direction of the current each time the loop reaches the vertical position, which as we know would change the direction of the dipole vector. This can be accomplished by using alternating current as the source current, resulting in what is referred to as an AC motor. However, sustained rotation can also be achieved with direct current by using what are called commutators and brushes. These devices attach to the shaft and allow the current to switch direction without having to directly change the supply current. This is referred to as a DC motor.

__Final Summary for Magnetism Introduction__

Magnets
• Materials that show strong magnetic effects are referred to as ferromagnetic. • Magnets have a north and south pole that follow similar attractive and repulsive laws that electric charges do, mainly; like poles attract and unlike poles repel. • Isolated magnetic poles have not so far been found to exist. • Magnetic field lines emerge from the north pole and terminate at the south pole and always form closed loops.
Magnetic Force on a Moving Charge
When a charged particle moves through a magnetic field, $\boldsymbol{B}$**,** a magnetic force acts on the particle and is given by: $$\boldsymbol{F}_B = q\boldsymbol{v}\; x\; \boldsymbol{B}$$ Where, q is the particles charge, and $\boldsymbol{v}$ is the particles velocity. The direction of $\boldsymbol{F}_B$ is given by the right-hand rule and the magnitude is found as follows. $$F = \|q\|vB\sin(\theta)$$ Where, θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{B}$, or we can say that $B\sin(\theta)$ is the component of the magnetic field that is perpendicular to the motion of the particle.
Magnetic Force on a Current Carrying Wire
When a current carrying wire is placed in a magnetic field, $\boldsymbol{B}$**,** a magnetic force acts on the wire and is given by: $$\boldsymbol{F}_B = i\boldsymbol{L}\; x\; \boldsymbol{B}$$ Where i is the current, and $\boldsymbol{L}$, is a vector in the direction of the current and with magnitude of the length of the wire. The magnitude of the magnetic force is found as follows: $$F_B = iLB\sin(\theta)$$ Where, θ is the angle between $\boldsymbol{L}$ and $\boldsymbol{B}$, or we can say that $B\sin(\theta)$ is the component of the magnetic field that is perpendicular to the direction of the current.
Torque on a Current Loop
A current loop placed in a magnetic field, $\boldsymbol{B}$, will experience a torque given by: $$\boldsymbol{\tau} = \boldsymbol{u}\; x\; \boldsymbol{B}$$ Where, $\boldsymbol{u} = Ni\boldsymbol{A}$**.** N is the number of loops, i is the current, and $\boldsymbol{A}$ is a vector that points in a direction that is perpendicular to the plane of coil and consistent with the right-hand rule (wrap your fingers in the direction of the current and your thumb points in the direction of the vector). The magnitude of $\boldsymbol{A}$ is equal to the area of the loop.

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