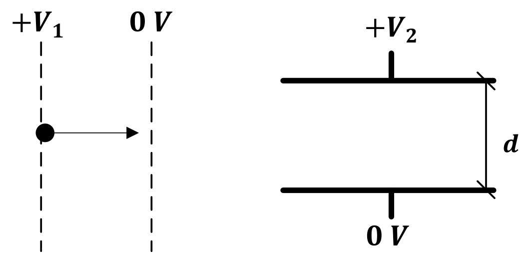
Magnetism Examples

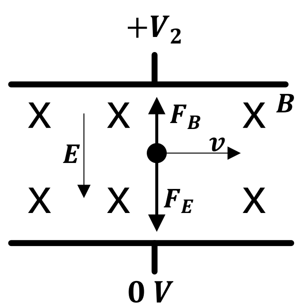
Question 1:

A proton is accelerated from rest through a potential difference of $1000\ V$. The proton then enters the gap between 2 parallel plates with a separation distance of $0.01\ m$ and a potential difference of $100\ V$ as shown in the figure below. What are the direction and magnitude of the magnetic field that is required to keep the proton moving in a straight line?



Solution 1:

When the proton enters the region between the two parallel plates it will experience an electric force directed downward. The magnetic force will need to counteract this force so that the net force is equal to zero. By the right-hand rule the magnetic field should be directed into the page so that that the resulting magnetic force is upward. The electric and magnetic field along with the forces are shown below.



To determine the magnitude of the required magnetic field we set the electric and magnetic forces equal.

$$F_B = F_E$$
$$\cancel{q}vB = \cancel{q}E$$
$$B = E\frac{1}{v}$$
$$B = \left(\frac{V_2}{d}\right)\cdot\frac{1}{v}$$

Next, to find the velocity of the proton as it enters the second region we use conservation of energy. The proton is initially at rest with a potential energy, U.

$$U = qV_1$$

At the end of the first region this potential energy has been fully converted to kinetic energy, therefore we have.

$$\frac{1}{2}mv^2 = qV_1$$

$$v = \sqrt{\frac{2qV_1}{m}}$$

We can now substitute this result into the magnetic field equation from above and solve for B.

$$B = \frac{V_2}{d} \cdot \frac{1}{\sqrt{\frac{2qV_1}{m}}}$$

$$B = \frac{V_2}{d} \cdot \sqrt{\frac{m}{2qV_1}}$$

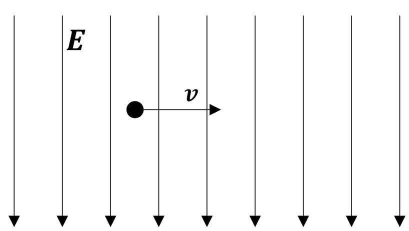
$$B = \frac{100}{0.01} \cdot \sqrt{\frac{1.673\, E^{-27}}{2 \cdot 1.6\, E^{-19} \cdot 1000}}$$

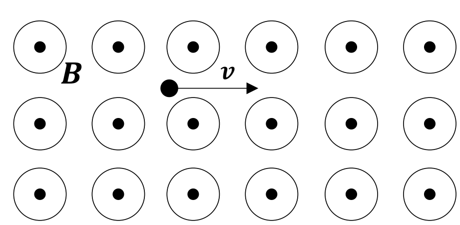
$$B = 0.023\, T$$

Question 2:

A proton is traveling in a straight line with a speed of $2.0\, E^6\ m/s$.

a.) Describe the change in its path when an electric field with $E = 10\, KV/m$ is activated.
b.) Describe the change in its path when a magnetic field with $B = 0.2\, T$ is activated.





Solution 2:

Part a.

The electric field, once activated, applies a constant force in the negative y direction. Therefore, the proton will accelerate downward while maintaining a constant speed in the x direction. We can start our analysis by finding the acceleration using Newton's 2nd law.

$$F_E = m(-a_y)$$
$$qE = m(-a_y)$$
$$a_y = -\frac{qE}{m}$$

Next, using kinematics, we find the x and y positions as functions of time.

$$y(t) = \cancel{y(0)} + \cancel{v_y(0)t} + \frac{1}{2}a_y t^2$$
$$y(t) = -\frac{qE}{2m}t^2$$
$$y(t) = -\frac{1.6E^{-19} \cdot 10E^3}{2 \cdot 1.673E^{-27}}t^2$$
$$y(t) = -4.78E^{11}t^2$$

$$x(t) = \cancel{x(0)} + v_x(0)t + \cancel{\frac{1}{2}a_x t^2}$$
$$x(t) = v_x t$$
$$x(t) = 2E^6 t$$

Note that by solving for t in the x equation and substituting into the y equation, we see that the proton follows a parabolic path in the x-y plane.

$$y(x) = -0.1195x^2$$

Part b.

The magnetic field, once activated, applies a force that is constant in *magnitude only*, since we know the direction of the magnetic force is given by the right-hand rule and will always be perpendicular to the velocity. A particle undergoing this type of force, which should be familiar to us from our earlier studies, will be in uniform circular motion. We label the force as a radial force and write the uniform circular motion equation.

$$F_r = ma_r$$
$$F_r = m\frac{v^2}{r}$$

In this case the radial force is supplied by the magnetic force. Substituting the magnetic force for F_r and solving for the radius we have

$$
\begin{aligned}
q\cancel{v}B &= \frac{mv^{\cancel{2}}}{r} \\
r &= \frac{mv}{qB} \\
r &= \frac{1.673E^{-27} \cdot 2E^{6}}{1.6E^{-19} \cdot 0.2} \\
r &= 0.1046\,m
\end{aligned}
$$

We can also solve for the period as shown below.

$$
\begin{aligned}
T &= \frac{2\pi r}{v} \\
T &= \frac{2\pi \cdot 0.1046}{2E^{6}} \\
T &= 0.3285\,\mu s
\end{aligned}
$$

For additional insight let's determine the time for the proton to move in the negative y direction a distance of $2r$ in both cases. The path of the proton in the magnetic field is a circle of radius r, for which we found the period, T, above. The first time the particle reaches the bottom of the circle would then be:

$$T/2 = 0.16425\,\mu s$$

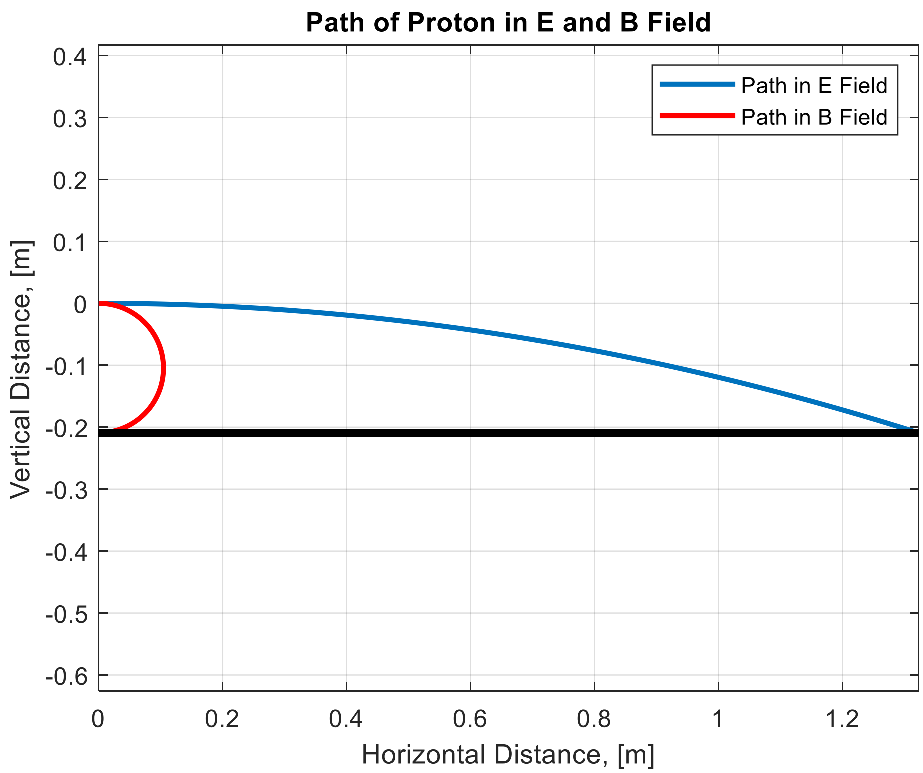
On the other hand, the proton follows a parabolic path in the electric field. In this case we can find the time for the y coordinate to reach $-2r$ as follows:

$$
\begin{aligned}
y(t) &= -4.78E^{11}t^{2} \\
-2r &= -4.78E^{11}t^{2} \\
t &= \sqrt{\frac{-2 \cdot 0.1046}{-4.78E^{11}}} \\
t &= 0.662\,\mu s
\end{aligned}
$$

Note, however in this case the proton has also moved horizontally a distance of

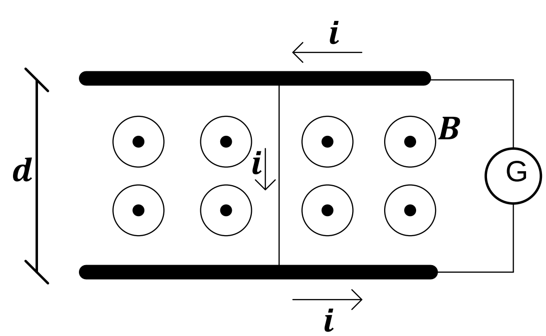
$$
\begin{aligned}
x &= 2E^{6} \cdot 0.662E^{-6} \\
x &= 1.32\,m
\end{aligned}
$$

Finally, to illustrate we show the path of the proton in both cases below as it moves an equal distance in the y direction.



Question 3:

A metal wire of mass $m = 0.025\ kg$ is attached on two horizontal rails separated by a distance of $0.03\ m$ and can slide with negligible friction. A current source, G, is attached to the rails and the entire apparatus is placed in a magnetic field with a strength of $B = 0.06\ T$ as shown below. The current source is turned on at $t = 0$ and produces a constant current of $0.01\ A$. Find the speed and direction of motion of the wire at $t = 61\ ms$.



Solution 3:

By the right-hand rule we can see that the force on the wire is pointing towards the left. There is also a force on the rails directed outward, but we can assume the rails are held in place. Since the only force on the wire is due to the magnetic field we find the acceleration using Newtons 2nd law.

$$F_B = ma$$
$$idB = ma$$
$$a = \frac{idB}{m}$$

Since the acceleration is constant the velocity is given by:

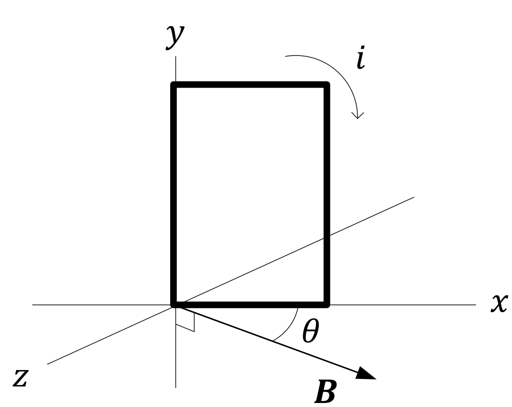
$$v(t) = v(0) + at$$
$$v(t) = \frac{idB}{m}t$$

And the velocity at 61 ms is

$$v(0.061) = \frac{0.01 \cdot 0.03 \cdot 0.06}{0.025} \cdot 0.061$$
$$v(0.061) = 4.4E^{-5}\ m/s$$

Question 4:

The figure below shows a rectangular 20 turn coil of wire with dimensions $0.1\ m$ by $0.05\ m$. It carries a current of $0.10\ A$ in the direction shown and is hinged along the y axis from one side. A $0.5\ T$ magnetic field is oriented at $\theta = 30^{\circ}$ in the x-z plane. What is the magnitude and direction of the torque acting on the coil?



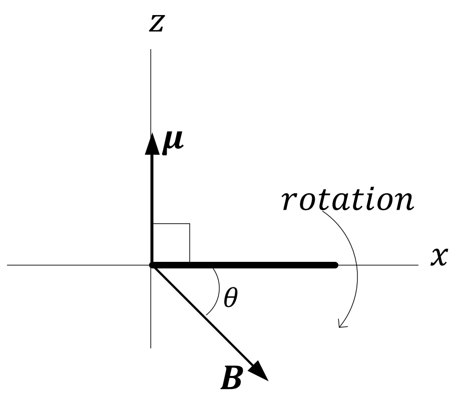
Solution 4:

To determine the direction of the torque we need to look at the magnetic dipole vector as compared to the magnetic field vector, since as you'll recall the torque will act to align the dipole vector with the direction of the field. The figure below is looking down in the x-z plane and shows the torque will act to rotate the coil clockwise about the y axis. The magnitude of the torque is given as follows:

$$\tau = NiAB\sin(\theta)$$

$$\tau = 20 \cdot 0.10 \cdot (0.1 * 0.05) \cdot 0.5\sin(90 + 30)$$

$$\tau = 0.00433\ N \cdot m$$



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