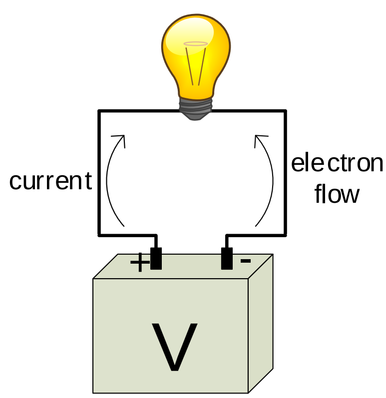
Resistor Circuits Introduction

Until now we have been discussing static electric charges. Recall however when introducing the capacitor, we saw that there is a short period of time after we apply a voltage source that charge flows onto the plates of the capacitor. We call this flow of charge *electric current*. Most of us are familiar with electric current flowing through the wires in our homes and used for types of electric appliances and devices. The most basic use that usually comes to mind when thinking of electric current is the light bulb. A conventional incandescent light bulb "glows" when electric current flows through a thin wire filament inside the glass bulb. Of course, there are many other examples of using electric current from the heating element in your toaster to the much more complex operation of your cell phone or other electronic devices. For electric current to flow we need a source of electric charges. The electric battery, invented in 1800 by Alessandro Volta, can serve as our source of a steady flow of electric charges. Without describing in detail, we note that an electric battery produces an electric current via various chemical reactions.

Electric Current:

As shown below, an electric potential (voltage) exist between the two terminals of a battery. When these two terminals are connected to a continuous path of conducting material (e.g. metal wire), an electric current will flow. Although physically negatively charged electrons flow from the negative terminal to the positive terminal, by convention (which was adopted before the discovery of the electron) electric current is defined by imagining positive charges flowing in the opposite direction. To make use of the current flowing through the wire we place a device (e.g. light bulb) in this path.



The electric current is defined as the net amount of charge passing through a cross section of the wire per unit time. It is measured in coulombs per second, which is given the name Ampere (A), after the French physicist Andre Ampere.

$$I = \frac{dQ}{dt}$$

Resistance:

As we mentioned an electric current is created in a wire when a potential difference is placed between two ends of a closed path. It was established experimentally that the amount of electric current is directly proportional to the potential difference applied.

$$I \propto V$$

In other words, the current flowing through a wire when we connect is 12-volt battery would be twice as much that would flow from a 6-volt battery. The proportionality constant can also be found experimentally for various sizes and types of wires and devices. With this constant known we can write the following relationship.

$$I = GV$$

Where G, defined as the *conductance*, describes the ease at which charge can flow through a wire or device when a potential difference is applied.

This relationship, however is conventionally written using the inverse of conductance, which we define as *resistance*.

$$R = \frac{1}{G}$$

The resistance then describes the *opposition* to electric current, and the relationship that some of us may already be familiar with, called Ohm's law, is written as follows:

$$V = IR$$

Where R is known as the resistance and is given the unit Ohm (Ω), after George Simon Ohm who was the first to establish this relationship for metal wires.

As an example, what would the resistance be of a small flashlight bulb connected to a 1.5-volt battery that draws 300 mA? Further, how much current would flow if we then used a 9-volt battery with the same flashlight bulb?

The resistance of the bulb is first determined as follows:

$$R = \frac{V}{I} = \frac{1.5}{0.30} = 5\ \Omega$$

Now that we know the resistance of the bulb we can find the current when we use a 9-volt battery.

$$I = \frac{V}{R} = \frac{9}{5} = 1.8\ A$$

Resistivity:

Further investigation reveals that for a given type of metal the resistance of a wire is proportional to the length and inversely proportional to the cross-sectional area.

$$R = \rho \frac{l}{A}$$

Where the proportionality constant, ρ, is called the *resistivity* and is material dependent.

As an example, suppose you wanted to connect a certain device that was $20\ m$ away using a copper wire ($\rho = 1.68\ E^{-8}$). What diameter of the wire should you purchase if you were required to keep the resistance below $0.1\ \Omega$?

The diameter of the cross section of a cylindrical wire is related to the area by:

$$A = \frac{\pi}{4} d^2$$

Solving the first equation for A and substituting the area formula above we can find the required diameter.

$$A = \rho \frac{l}{R}$$

$$\frac{\pi}{4} d^2 = \rho \frac{l}{R}$$

$$d = \sqrt{\rho \frac{4l}{\pi R}}$$

$$d = \sqrt{1.68\ E^{-8} \frac{4 \cdot 20}{\pi \cdot 0.1}}$$

$$d = 2.1\ mm$$

Electric Power:

We already know that any object in motion contains kinetic energy. Therefore, charges in motion can be said to contain electrical kinetic energy. This *electrical* energy is extremely useful as it can be easily transformed into other types of energy (e.g. light, thermal, sound, etc..). From our earlier lesson on electric potential we know that we can relate the electrical potential energy and electric potential through a charge element, dq as:

$$dU = dqV$$

As power is defined as energy per unit time, dividing both sides by dt we can derive an expression for the electric power.

$$\frac{dU}{dt} = \frac{dq}{dt}V$$

$$P = IV$$

This relationship allows us to determine the rate at which electric energy is delivered to a device given we know the voltage across that device as well as the current flowing through it. The unit for electric power is the watt $(1\ W = 1\ J/s)$. This relationship can also be written in two other ways using Ohm's law ($V = IR$).

$$P = IV$$

$$P = I(IR) = I^2R$$

$$P = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

As an example, find the power required to run an electric heater that draws 15 A when connected to a 120-volt source.

$$P = 15 \cdot 120$$

$$P = 1800\ W = 1.8\ kW$$

To determine the total amount of energy used to run the electric heater we can multiply this value by the amount of time it was used.

$$P = \frac{E}{\Delta t}$$

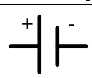
$$E = P\Delta t$$

Therefore, instead of using Joules as our unit for energy we can also use "watt-time". Electric companies most commonly use kilowatt-hours to charge their customers. A typical electric company may quote the rate as 12 cents per kilowatt-hour. Using this rate let's determine how much it would cost to run the heater above for 10 hours a day for 30 days.

$$\left(\frac{\$0.12}{1\ kWh}\right) \cdot\ (1.8\ kW) \cdot \left(10\frac{h}{d}\right) \cdot (30\ d) = \$64.80$$

Basic Resistor Circuits:

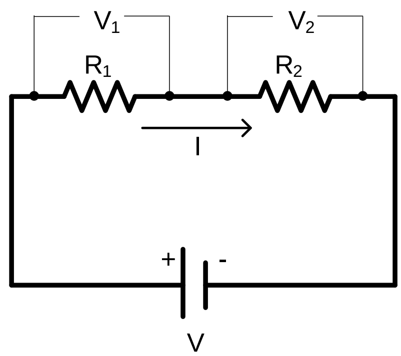
Electrical circuits are ubiquitous in society today. For example, throughout every home in modern societies electric circuits exist to supply electricity to the various appliances and lighting. In addition, much more complex circuits are present in the many electronic devices that are used today such as radios, TVs, computers, cell phones, etc. Any electric circuit requires a source of energy, such as a battery, and the most basic electric circuit contains a single battery and a device that can be described by its resistance (e.g. a light bulb). Electronic circuits on the other hand, are generally much more complex and contain various components, such as transistors, diodes, etc. which used to control the flow of the current. In electronic circuits it may be necessary to limit the flow of electricity through a certain part of a circuit. Components called *resistors* are used for this purpose. In this section we will examine circuits that contain only resistive components, whether they be devices that can be described by their resistance (e.g. light bulbs), or the physical components called resistors that are mostly used in various electronic devices. Just as we did with capacitor circuits from the last section we use certain symbols to represent the various components in a circuit. The table below shows the various symbols used in this section.

Battery	Resistor	Wire with negligible resistance
 V		

Just as we had with capacitors there are two basic ways we can connect resistors to each other in a circuit; in series or in parallel.

Series Connections:

Below is an electrical circuit diagram showing two resistors connected in series.



Let's look closer at the current and voltages for this circuit. Since no charge accumulates on either of the resistors, the same amount of current that passes through the first resistor will also pass through the second resistor.

$$I_1 = I_2 = I$$

For explanatory purposes let's assume the positive battery terminal is at potential $+V$, and the negative side is at 0 potential. Next, imagine traveling around the circuit starting at the positive battery terminal and ending at the negative terminal. Since the potential at the negative terminal is zero, (as is the potential on the right side of R_2), the potential must be decreased by some amount through each resistor such that: $V - V_1 - V_2 = 0$. Rearranging we have the following equation for voltages of resistors in series.

$$V = V_1 + V_2$$

Now using Ohm's law, we have the following.

$$V = I_1 R_1 + I_2 R_2$$
$$V = I(R_1 + R_2)$$
$$V = I R_{eq}$$

Where:

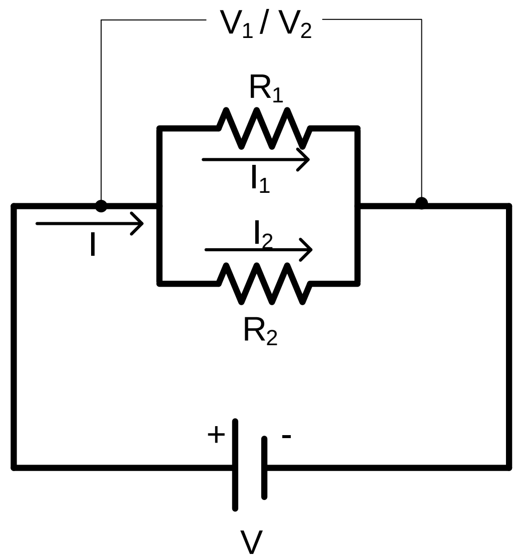
$$R_{eq} = (R_1 + R_2)$$

Therefore, replacing the two resistors from the above circuit with a single resistor with a value of R_{eq}, will result in the same circuit behavior. The equation holds for any number of resistors on series.

$$R_{eq,series} = (R_1 + R_2 + \cdots + R_N)$$

Parallel Connections:

Below is an electrical circuit diagram showing two resistors connected in parallel.



In this case the same voltage is across both resistors (i.e. $V_1 = V_2 = V$), however the total current leaving the battery enters the junction as shown and is divided between the two resistor paths, which allows us to write the following:

$$I = I_1 + I_2$$

We can now again use Ohm's law to find the equivalent resistance.

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$
$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$V = I\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Where:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

In this case the equivalent resistance for any number of resistors in parallel is given as follows:

$$R_{eq,parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1}$$

Kirchhoff's Rules for Solving Circuits:

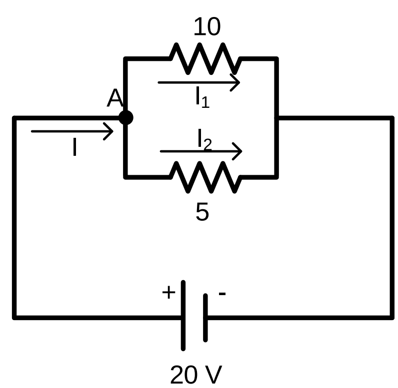
We can use the knowledge from above on how to combine resistors in series and parallel to find the currents flowing as well as the voltages across the different resistors for many basic circuits. However, for more complex circuits this technique may not be possible. Fortunately, Gustav Kirchhoff devised two fundamental rules that can be used to help find currents and voltages in more complicated electrical circuits. The rules are based on the much more fundamental principles of the conservation of electrical charge and the conservation of energy. We first state the rules below and illustrate each using a basic single source, single loop circuit. Finally, we end this section with one example on how these rules can be used for a more complex multi-source, multi-loop circuit.

Kirchhoff's Current Rule:

Kirchhoff's current rule, sometimes called "Kirchhoff's junction rule" may be stated as follows:

Kirchhoff's Current Rule:
The sum of the currents entering a junction must equal the sum of the currents leaving that same junction.

To illustrate consider a basic single loop circuit with two resistors in parallel as shown below.



Applying the current law at Junction A we have:

$$I = I_1 + I_2$$

Furthermore, as the voltage across each resistor is the same we can find the individual currents.

$$I_1 = \frac{20}{10} = 2\ A \qquad\qquad I_2 = \frac{20}{5} = 4\ A$$

Substituting into the Kirchhoff's current rule we can easily find the total current without having to resort to finding the equivalent resistance.

$$I = 2 + 4 = 6\ A$$

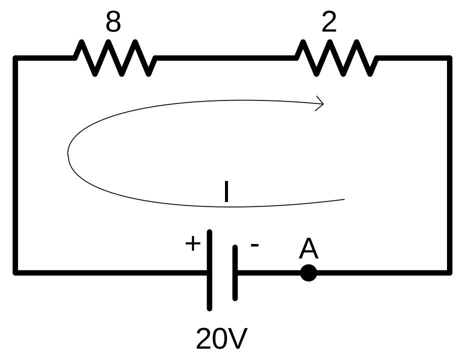
Note: We can verify the above result using $R_{eq} = \left(\frac{1}{10} + \frac{1}{5}\right)^{-1} = \frac{10}{3}$, and $I = \frac{V}{R_{eq}} = \frac{20}{10/3} = 6\ A$

Kirchhoff's Voltage Rule:

Kirchhoff's voltage rule, sometimes called "Kirchhoff's loop rule" may be stated as follows:

Kirchhoff's Voltage Rule:
The sum of the voltages around any closed loop in a circuit must equal zero.

To illustrate, consider a basic single loop circuit with two resistors in series as shown below.



The following steps outline the process for applying the voltage law around a single loop.

- Find a closed loop in the circuit and choose a direction for the current.
    - Note if choose the "wrong" direction we will end up with a negative value for the current, which tells us it travels in the opposite direction.
- Choose a single point in this loop and traverse the loop returning to this point.
    - In this case we choose point A.
- As you traverse the loop we note the change in potential through each component we pass.
    - The potential increases when we pass a battery from the negative terminal to the positive terminal.
    - The potential drops when we pass through a resistor in the direction of the current.
- Set the result to zero since when we arrive back at our starting point as the change in potential should be zero.

$$+V - IR_1 - IR_2 = 0$$

$$I = \frac{V}{R_1 + R_2}$$

$$I = \frac{20}{10} = 2A$$

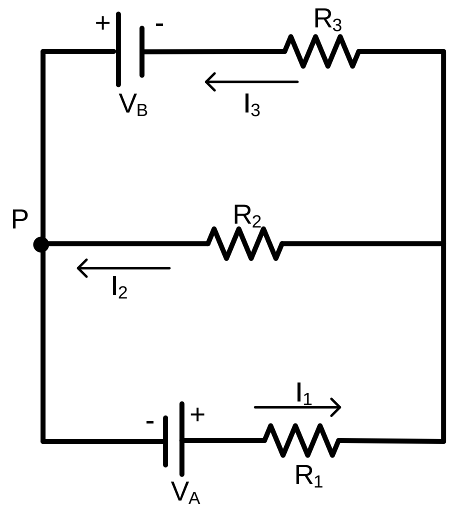
The voltage across each resistor can now easily be found.

$$V_1 = 2 \cdot 8 = 16\,V \qquad V_2 = 2 \cdot 4 = 4\,V$$

Note: The voltage rule applied to a single-loop circuit is rather straightforward, however you will see the utility with the multiloop example below.

Multiloop Circuit Example:

The utility of Kirchhoff's Rules is well illustrated with more complicated multiloop circuits. Let's solve for all currents and voltages in the multiloop circuit below.



$$V_A = 15\ V$$
$$V_B = 5\ V$$
$$R_1 = 5\ \Omega$$
$$R_2 = 20\ \Omega$$
$$R_3 = 10\ \Omega$$

Starting with Kirchhoff's current rule at junction P we have:

$$I_2 + I_3 = I_1$$

Next, we can use Kirchhoff's voltage law to write equations for the top and bottom loop.

Top Loop

$$-I_3R_3 + V_B + I_2R_2 = 0$$

Bottom Loop

$$V_A - I_1R_1 - I_2R_2 = 0$$

We now have 3 equations and three unknown currents, which we can solve with a little algebra.

1. Substitute for I_1 in the bottom loop equation from the current junction equation and then solve for I_2.

$$V_A - I_2R_1 - I_3R_1 - I_2R_2 = 0$$
$$I_2(R_1 + R_2) = V_A - I_3R_1$$
$$I_2 = \frac{V_A - I_3R_1}{(R_1 + R_2)}$$

2. Substitute this result into the top loop equation and solve for I_3.

$$-I_3R_3 + V_B + \frac{(V_A - I_3R_1)}{(R_1 + R_2)}R_2 = 0$$

$$-I_3R_3 + V_B + \frac{V_AR_2}{(R_1 + R_2)} - \frac{I_3R_1R_2}{(R_1 + R_2)} = 0$$

$$I_3\left(R_3 + \frac{R_1R_2}{(R_1 + R_2)}\right) = V_B + \frac{V_AR_2}{(R_1 + R_2)}$$

$$I_3 = \frac{V_B + \frac{V_AR_2}{(R_1 + R_2)}}{\left(R_3 + \frac{R_1R_2}{(R_1 + R_2)}\right)}$$

As the left side of this equation are all known quantities we can solve for I_3!

$$I_3 = \frac{5 + \frac{15 \cdot 20}{(5 + 20)}}{\left(10 + \frac{5 \cdot 20}{(5 + 20)}\right)} = 1.214\ A$$

3. Substitute this result into the top loop equation and solve for I_2.

$$I_2 = \frac{I_3R_3 - V_B}{R_2}$$

$$I_2 = \frac{1.214 \cdot 10 - 5}{20} = 0.357\ A$$

4. Finally, we can now use the current rule to solve forI_1.

$$I_1 = I_2 + I_3$$

$$I_1 = 0.357 + 1.214$$

$$I_1 = 1.571\ A$$

As a final step, since we now know all the currents, we can solve for the voltage drop across all three resistors.

$V_1 = I_1R_1$	$V_2 = I_2R_2$	$V_3 = I_3R_3$
$V_1 = 1.571 \cdot 5$	$V_2 = 0.357 \cdot 20$	$V_3 = 1.214 \cdot 10$
$V_1 = 7.857\ V$	$V_2 = 7.143\ V$	$V_3 = 12.143$

Final Summary for Resistor Circuits

Electric Current

An *electric current* is defined as the net amount of charge passing through a cross section of wire per unit time.

$$I = \frac{dQ}{dt}$$

Unit of measure is: $\frac{Coulombs}{second} \stackrel{\text{def}}{=} Ampere\ (A)$

Resistance / Ohm's Law

The *resistance, R,* describes the opposition to electric current and is measured in Ohms (Ω)

Ohm's Law describes the relationship between the current and voltage.

$$V = IR$$

Resistivity

For a given type of metal wire the resistance of is proportional to the length and inversely proportional to the cross-sectional area.

$$R = \rho \frac{l}{A}$$

Where, l is the length of the wire, A is the cross-sectional area, and the proportionality constant, ρ, is called the *resistivity*, which is material type.

Electric Power

Electric Power is defined as energy per unit time. We can use three different relationships to find the power for various circuit components.

$P = IV$ $\qquad$ $P = I^2R$ $\qquad$ $P = \frac{V^2}{R}$

Unit of measure is: $\frac{Joules}{second} \stackrel{\text{def}}{=} Watt\ (W)$

Resistor Circuits

Equivalent Resistance for Resistors in Series

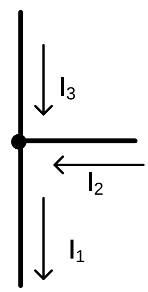
$$R_{eq,series} = (R_1 + R_2 + \cdots + R_N)$$

Equivalent Resistance for Resistors in Parallel

$$R_{eq,parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1}$$

Kirchhoff's Current Rule:

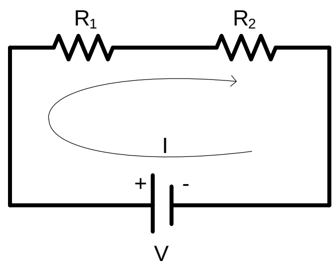
The sum of the currents entering a junction must equal the sum of the currents leaving that same junction.



$$I_1 = I_2 + I_3$$

Kirchhoff's Voltage Rule:

The sum of the voltages around any closed loop in a circuit must equal zero.



$$V - IR_1 - IR_2 = 0$$

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