Resistor Circuit Examples

Question 1:

A certain piece of copper wire of length 2 meters has a resistance of 10 Ohms. You are told to cut the wire into two pieces such that one piece has 5 times the resistance of the other. Determine where to cut the wire and what the resistance of the resulting two pieces are.

Solution 1:

The resistance of the two pieces of the wire can be expressed as:

$$R_1 = \rho\frac{l_1}{A}, \quad R_2 = \rho\frac{l_2}{A}$$

The first requirement is that $R_1 = 5R_2$, therefore:

$$\cancel{\rho}\frac{l_1}{\cancel{A}} = \cancel{\rho}\frac{5l_2}{\cancel{A}}$$
$$l_1 = 5l_2$$

Another requirement is that $l = l_1 + l_2$, therefore we can write the following:

$$5l_2 + l_2 = l$$
$$l_2 = \frac{l}{6}$$

Therefore, the lengths of the two pieces are given below.

$$l_1 = \frac{5l}{6} = \frac{5 \cdot 2}{6} = 1.67\ m \qquad\qquad l_2 = \frac{l}{6} = \frac{2}{6} = 0.33\ m$$

The resistance of each piece can now be found.

$$R_1 = \rho\frac{l_1}{A} = \rho\frac{5l}{6A} = \frac{5}{6}\left(\rho\frac{l}{A}\right) = \frac{5}{6}R \qquad\qquad R_2 = \rho\frac{l_2}{A} = \rho\frac{l}{6A} = \frac{l}{6}\left(\rho\frac{l}{A}\right) = \frac{1}{6}R$$
$$R_1 = 8.33\ \Omega \qquad\qquad R_2 = 1.67\ \Omega$$

Question 2:

Some households are replacing older incandescent light bulbs with new LED light bulbs. A certain LED bulb is said to provide the same lighting as a 100-watt incandescent bulb using only 14 watts. a.) Find the resistance and current for each bulb assuming a 120-volt source. b.) Find the yearly cost to keep 10 light bulbs on for 8 hours a day if the electric company is charging 0.12 cents per kWh.

Solution 2:

Part a.)

The resistance and current for each bulb type can be found as follows.

	Incandescent Bulb	***LED Bulb***
Resistance	$R = \frac{V^2}{P}$ $R = \frac{120^2}{100} = 144\ \Omega$	$R = \frac{V^2}{P}$ $R = \frac{120^2}{14} = 1029\ \Omega$
Current	$I = \frac{P}{V}$ $I = \frac{100}{120} = 0.83\ A$	$I = \frac{P}{V}$ $I = \frac{14}{120} = 0.12\ A$

Part b.)

The yearly cost can be found as below.

Incandescent Bulb	$\left(\frac{\$0.12}{kWh}\right) \cdot (10 * 100\ W) \cdot \left(\frac{1\ kW}{1000\ W}\right) \cdot \left(8 \frac{h}{d}\right) \cdot \left(365\ \frac{d}{y}\right) = \$350.40$
LEB Bulb	$\left(\frac{\$0.12}{kWh}\right) \cdot (10 \cdot 14\ W) \cdot \left(\frac{1\ kW}{1000\ W}\right) \cdot \left(8 \frac{h}{d}\right) \cdot \left(365\ \frac{d}{y}\right) = \$49.06$

Question 3:

If a voltage source is applied to N resistors in series, show that the source voltage is split across each resistor based on the ratio of the resistance of that resistor to the total resistance of the circuit.

If a voltage source is applied to N resistors in parallel, show that the total current is split between each resistor based on ratio of the conductance of that resistor to the total conductance of the circuit.

Solution 3:

Series Circuit (Voltage Divider):

The same current, I, flows through resistors in series.

$$I = \frac{V}{R_{eq}}$$

And the voltage across the n^{th} resistor is:

$$V_n = IR_n$$

Which upon substitution gives the result below.

$$V_n = V\left(\frac{R_n}{R_{eq}}\right)$$
$$V_n = V\left(\frac{R_n}{\sum_{n=1}^{N} R_n}\right)$$

This result can be used to find the voltage across any resistor in a circuit where all resistors are in series, sometimes called a voltage divider circuit.

Parallel Circuit (Current Divider):

Recall the conductance, G, is the reciprocal of the resistance, $G = \frac{1}{R}$, and therefore, $I = GV$. Furthermore, the equivalent conductance for resistors in parallel can be derived as follows:

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_1} + \cdots + \frac{1}{R_1}\right)$$
$$G_{eq} = (G_1 + G_1 + \cdots + G_1)$$

The total current flowing, I_T, in the circuit is related to the voltage as below.

$$V = \frac{I_T}{G_{eq}}$$

And the current through the n^{th} resistor is:

$$I_n = G_n V$$

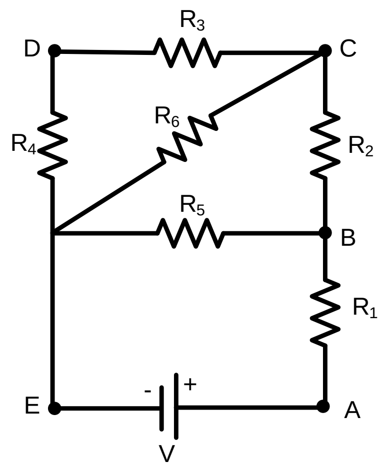
Which upon substitution gives the result below.

$$I_n = I_T \frac{G_n}{G_{eq}}$$
$$I_n = I_T\left(\frac{G_n}{\sum_{n=1}^{N} G_n}\right)$$

This result can be used to find the current though any resistor in a circuit where all resistors are in parallel, sometimes called a current divider circuit.

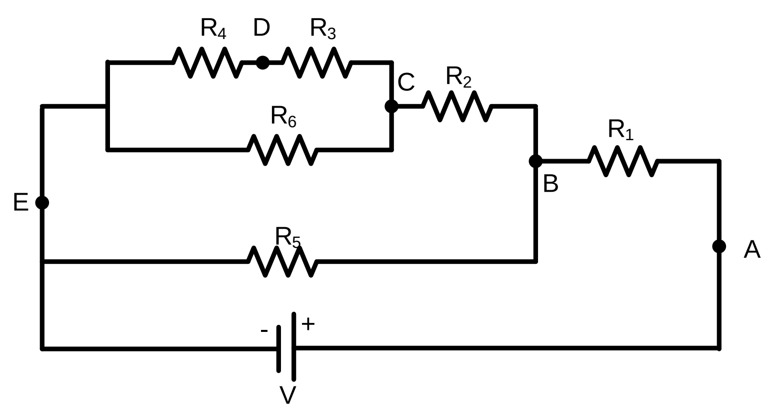
Question 4:

Find the equivalent resistance of the circuit below. Each resistor has a value of $2\ k\Omega$.



Solution 4:

By carefully redrawing the circuit it can be made more obvious which resistors are in series and which are in parallel. Marking each junction point and labeling all resistors helps to be sure the redrawn circuit is equivalent.



With the redrawn circuit it should be clear which resistors are in series and which are in parallel.

We can start with the equivalent resistance of the top most branch as:

$$R_{34} = R + R = 2R$$

This resistance is in parallel with R_6, so we have:

$$R_{346} = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1}$$

$$R_{346} = \frac{2R}{3}$$

Next, we can add R_2 in series as follows.

$$R_{3462} = \frac{2R}{3} + R$$
$$R_{3462} = \frac{5R}{3}$$

Which is in parallel with R_5.

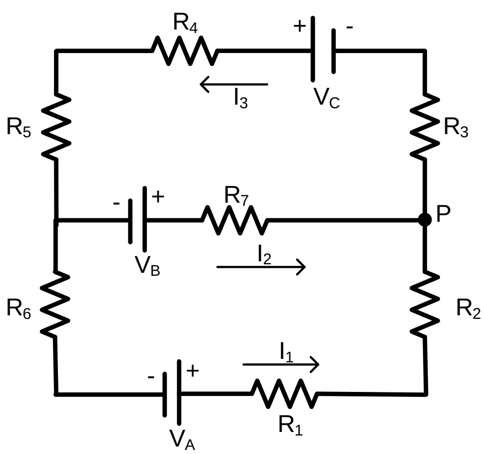
$$R_{34625} = \left(\frac{3}{5R} + \frac{1}{R}\right)^{-1}$$
$$R_{34625} = \frac{5R}{8}$$

Finally, this value is in series with R_1, so that the final equivalent resistance is computed as shown.

$$R_{eq} = \frac{5R}{8} + R$$
$$R_{eq} = \frac{13R}{8}$$
$$R_{eq} = \frac{13 \cdot 2000}{8} = 3250\ \Omega$$

Question 5:

Find the currents in the multiloop circuit below.



$V_A = 6\ V$

$V_B = V_c = 12\ V$

$R_1 = R_4 = 2\ \Omega$

$R_2 = 18\ \Omega$

$R_3 = 12\ \Omega$

$R_5 = 8\ \Omega$

$R_6 = 15\ \Omega$

$R_7 = 10\ \Omega$

Starting with Kirchhoff's current rule at junction P we have:

$$I_1 + I_2 = I_3$$

Next, we can use Kirchhoff's voltage law to write equations for the top and bottom loop.

Top Loop

$$V_C - I_3R_4 - I_3R_5 + V_B - I_2R_7 - I_3R_3 = 0$$
$$(V_C + V_B) - I_3(R_4 + R_5 + R_3) - I_2R_7 = 0$$
$$24 - 22I_3 - 10I_2 = 0$$

Bottom Loop

$$V_A - I_1R_1 - I_1R_2 + I_2R_7 - V_B - I_1R_6 = 0$$
$$(V_A - V_B) - I_1(R_1 + R_2 + R_6) + I_2(R_7) = 0$$
$$-6 - 35I_1 + 10I_2 = 0$$

We now have 3 equations and three unknown currents, which we can solve with a little algebra.

1. Substitute for I_1 in the bottom loop equation from the current junction equation and then solve for I_2.

$$-6 - 35(I_3 - I_2) + 10I_2 = 0$$
$$45I_2 = 35I_3 + 6$$
$$I_2 = \frac{35I_3 + 6}{45}$$

2. Substitute this result into the top loop equation and solve for I_3.

$$24 - 22I_3 - 10\left(\frac{35I_3 + 6}{45}\right) = 0$$
$$24 - 22I_3 - \frac{70}{9}I_3 - \frac{4}{3} = 0$$
$$\frac{268}{9}I_3 = \frac{68}{3}$$
$$I_3 = \frac{68 \cdot 9}{3 \cdot 268} = 0.76\ A$$

3. Substitute this result into the top loop equation and solve for I_2.

$$I_2 = \frac{24 - 22I_3}{10}$$
$$I_2 = \frac{24 - 22(0.76\,)}{10} = 0.73\ A$$

4. Finally, we can now use the current rule to solve for I_1.

$$I_1 = I_3 - I_2$$
$$I_1 = 0.76 - 0.73$$
$$I_1 = 0.03\ A$$

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