Capacitance Examples

Question 1:

Find the capacitance of a parallel plate capacitor that uses circular plates of radius 2 mm that are separated by 0.1 mm. Find the charge stored on the capacitor when it is connected to a 1.3-volt battery.

Solution 1:

We can use the expression we developed for the capacitance for parallel plate capacitors.

$$C = \epsilon_0 \frac{A}{d}$$
$$C = \epsilon_0 \frac{2\pi r}{d}$$
$$C = 8.8542E^{-12} \frac{2\pi \cdot 0.002}{0.0001}$$
$$C = 1.113\ nF$$

The charge is then found as follows:

$$Q = CV$$
$$Q = 1.113 \cdot 1.3$$
$$Q = 1.45\ nC$$

Question 2:

Let's start with the capacitor from question 1 and remove the battery once the capacitor is fully charged.

a.) What is the new voltage across the capacitor if we now double the distance between the plates? What is the change in the electrical potential energy stored in the capacitor?

b.) What is the new voltage if instead we placed an insulator between the plates with a dielectric constant of $K = 2$? What is the change in the electrical potential energy stored in the capacitor?

c.) What would happen if we did the same two experiments in a.) and b.) while leaving the battery connected?

Solution 2a:

We know the capacitance will change if we change the distance between the plates. However, we also know that since charge is conserved the same amount of charge will remain on the plates of the capacitor. Therefore, the voltage must change to satisfy the below fundamental relationship.

$$V = \frac{Q}{C}$$

The geometrical capacitance equation for the capacitor after doubling the distance is given as:

$$C_{new} = \epsilon_0 \frac{A}{2d} = \frac{C}{2}$$

Therefore, the new voltage is computed as follows:

$$V_{new} = \frac{Q}{C_{new}}$$
$$V_{new} = \frac{2Q}{C}$$
$$V_{new} = 2V$$
$$V_{new} = 2.6\ V$$

Note: Since the capacitance, (charge stored per volt), decreased, the new capacitor can hold less charge for a given voltage. And since the charge remained the same the voltage must increase.

The change in energy stored in the capacitor can be found as follows:

$$\Delta U = U_f - U_i$$
$$\Delta U = \frac{1}{2}QV_{new} - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}Q2V - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}QV(2-1)$$
$$U_f - U_i = U_i$$
$$U_f = 2U_i$$

Which shows that the final stored energy is twice what was initially stored.

Solution 2b:

The capacitance is again changed based on the dielectric as:

$$C_{new} = KC$$

The new voltage is again computed as follows:

$$V_{new} = \frac{Q}{C_{new}}$$
$$V_{new} = \frac{Q}{KC}$$
$$V_{new} = \frac{V}{K} = 0.65\ V$$

Note: In this case the capacitance, (charge stored per volt), increased, therefore the new capacitor can hold more charge for a given voltage. And again, since the charge remained the same the voltage could decrease.

Like what was done in part a.) we can find the final stored energy as a function of the initial energy.

$$\Delta U = U_f - U_i$$
$$\Delta U = \frac{1}{2}QV_{new} - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}Q\frac{V}{K} - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}QV\left(\frac{1}{K} - 1\right)$$
$$\Delta U = U_i\left(\frac{1}{2} - 1\right)$$
$$U_f - U_i = -\frac{1}{2}U_i$$
$$U_f = \frac{1}{2}U_i$$

In this case the final stored energy is half what was initially stored.

Solution 2c:

In this case we perform the same two experiments from above, but this time since the voltage remains the same the charge can change. Let's first find the charge on the capacitor when the separation distance is increased.

$$Q_{new} = C_{new}V$$
$$Q_{new} = \frac{1}{2}CV$$
$$Q_{new} = \frac{Q}{2} = 0.725\ nC$$

Note: Since the capacitance, (charge stored per volts), decreased, the new capacitor can hold less charge for a given voltage. And since the **voltage** remains the same the charge will decrease.

In this case the change in energy is:

$$\Delta U = U_f - U_i$$
$$\Delta U = \frac{1}{2}Q_{new}V - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}\frac{Q}{2}V - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}QV\left(\frac{1}{2} - 1\right)$$
$$U_f - U_i = -\frac{1}{2}U_i$$
$$U_f = \frac{1}{2}U_i$$

Note this is opposite to the case where we disconnected the battery first.

In the next experiment we place a dielectric between the plates with battery still connected.

$$Q_{new} = C_{new}V$$
$$Q_{new} = KCV$$
$$Q_{new} = 2Q$$
$$Q_{new} = 1.9\ nC$$

Note: In this case the capacitance, (charge stored per volt), increases, therefore the new capacitor can hold more charge for a given voltage. And again, since the **voltage** remains the same the charge will increase.

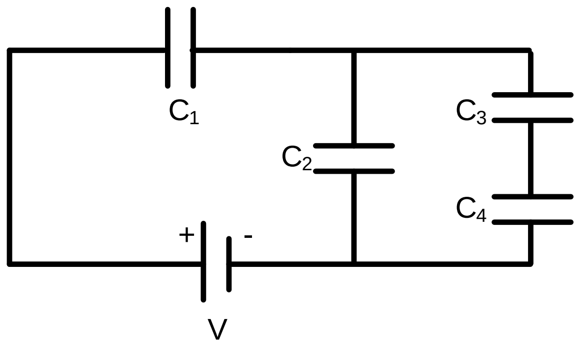
Finally, we once more find the change in stored energy.

$$\Delta U = U_f - U_i$$
$$\Delta U = \frac{1}{2}QV_{new} - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}2QV - \frac{1}{2}QV$$
$$\Delta U = \frac{1}{2}QV(2 - 1)$$
$$U_f - U_i = U_i$$
$$U_f = 2U_i$$

This again is opposite to the case where we disconnected the battery first.

Question 3:

Suppose $C_1 = C_2 = C_3 = C_4 = C$. a.) Find the equivalent capacitance. b.) Find the charge and voltage across each capacitor.

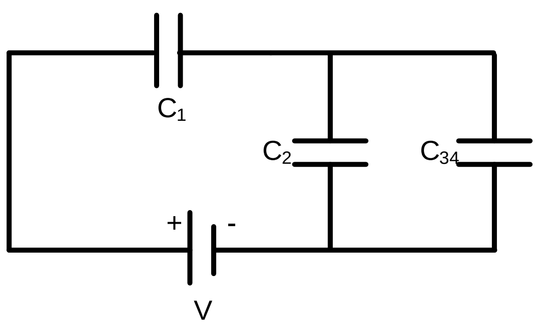


Solution 3a:

We can start by noticing that C_3 and C_4 are in series, therefore:

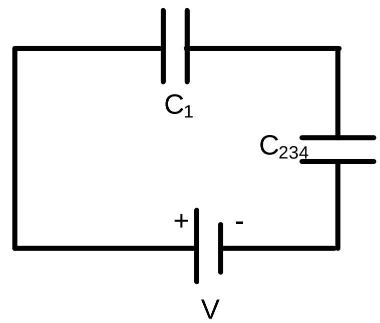
$$C_{34} = \left(\frac{1}{C_3} + \frac{1}{C_4}\right)^{-1}$$
$$C_{34} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1}$$
$$C_{34} = \frac{C}{2}$$

Redrawing we now see that C_2 and C_{34} are in parallel.



$$C_{234} = C_2 + C_{34}$$
$$C_{234} = C + \frac{C}{2}$$
$$C_{234} = \frac{3C}{2}$$

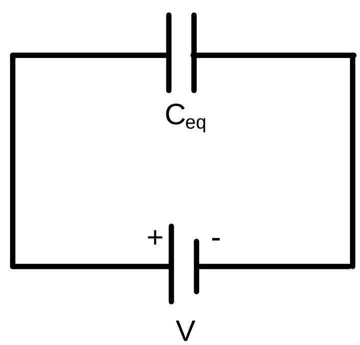
Redrawing again we now see that C_1 and C_{234} are in series.



$$C_{1234} = \left(\frac{1}{C_1} + \frac{1}{C_{234}}\right)^{-1}$$
$$C_{1234} = \left(\frac{1}{C} + \frac{2}{3C}\right)^{-1}$$
$$C_{1234} = \frac{3C}{5}$$

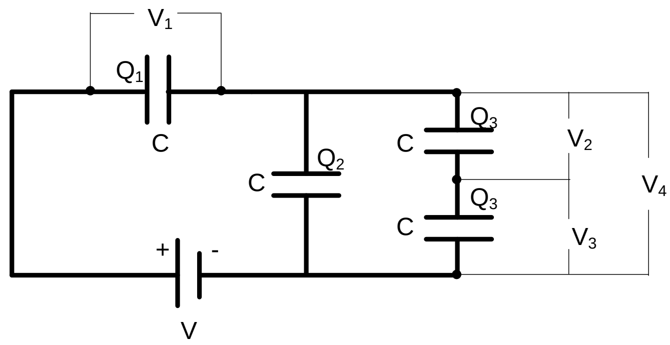
Finally, we can redraw the final circuit with a single equivalent capacitor, with:

$$C_{eq} = \frac{3C}{5}.$$

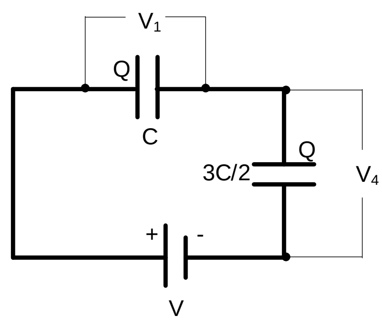


Solution 3b:

We first redraw the circuit showing all equal value capacitors along with the different voltages and charges. Note the same voltage is across capacitors in parallel and the same charge is on capacitors in series.



By combining the two parallel branches on the left we can start by finding V_1 and V_4 since the charges are the same.



We can write the following three equations for the above circuit and then follow the remaining steps below to solve for V1 and V4.

$$V = V_1 + V_4 \qquad V_1 = \frac{Q}{C} \qquad V_4 = \frac{2Q}{3C}$$

- Substitute equation 2 into equation 3.

$$V_4 = \frac{2V_1\cancel{C}}{3\cancel{C}}$$

$$V_4 = \frac{2V_1}{3}$$

- Substitute the result into equation 1 and solve for V_1

$$V_1 + \frac{2V_1}{3} = V$$

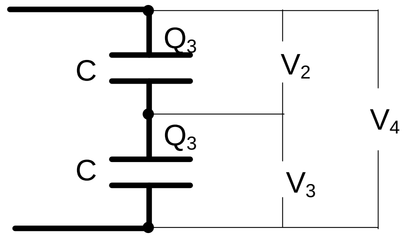
$$\frac{5V_1}{3} = V$$

$$V_1 = \frac{3}{5}V$$

Therefore:

$$V_4 = \frac{2}{5}V$$

Next, we can determine the voltage split across the rightmost two capacitors.



In this case the three equations are as follows:

$$V_4 = V_2 + V_3 \qquad V_2 = \frac{Q_3}{C} \qquad V_3 = \frac{Q_3}{C}$$

The last two equations tell us that $V_2 = V_3$, which means that V_4, which we found to be $\frac{2}{5}V$, is evenly split between the two capacitors.

$$V_2 = V_3 = \frac{1}{5}V$$

Lastly, the charge on each capacitor is easily determined since we know all voltages.

$$Q_1 = CV_1 \qquad Q_2 = CV_4 \qquad Q_3 = CV_2$$

$$Q_1 = \frac{3}{5}CV \qquad Q_2 = \frac{2}{5}CV \qquad Q_3 = \frac{1}{5}CV$$

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