Electric Potential Introduction - Part 1

As we have stated earlier when studying Newtonian mechanics, the conservation of energy principle is one the most important (and useful) concepts in physics. More specifically, the conservation of mechanical energy relates to systems where only conservative forces (e.g. gravitational) are acting.


<table>
<thead>
<tr>
<th>The Principle of the Conservation of Mechanical Energy</th>
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<tbody>
<tr>
<td>In a closed system where only conservative forces are acting; the kinetic and potential energy may change but the total mechanical energy remains constant.</td>
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<tr>
<td>$E_{mec} = K + U$</td>
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<tr>
<td>$U_i + K_i = U_f + K_f$</td>
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Since the gravitational force is conservative we can define the gravitational potential energy. The electrical force, as it has the same form as the gravitational force, is also conservative and therefore we can also define electrical potential energy. In this section we would like to define the electric potential, which is closely related to electrical potential energy. Of course, gravitational potential can also be defined, however, as we will see, the electric potential is a common quantity that most of us are familiar with in our daily lives.

We start by recalling that when work is done by a conservative force on an object, the kinetic energy increases and the potential energy decreases.

$W = \Delta K$ \hspace{1cm} $W = -\Delta U$

As we may be more familiar with gravitational potential energy, let’s start there before moving to electric potential energy.

**Gravitational Potential Energy Example:**

An object that is placed at a certain height above the earth, (i.e. in the gravitational field of the earth), without any external forces, will accelerate towards the earth due to the gravitational force. The work done on this object by the gravitational force is given as follows:

$W = F_g \Delta h$

Using the work-potential energy relationship from above we can express the change in potential energy as follows:

$\Delta U = -F_g \Delta h$
$\Delta U = -mg \Delta h$
**Electrical Potential Energy Example:**

Now if we place a *charged* object at a certain location in an electric field without any external forces, the object will accelerate in a direction according to the direction of the electrical force. The work done on the charged object is then similarly given as follows:

\[ W = F_e \Delta d \]

And again, we can express the change in potential energy as follows:

\[ \Delta U = -F_e \Delta d \]
\[ \Delta U = -qE \Delta d \]

Note that we show two cases in this example because, unlike with mass, a charged particle can be both positive and negative. In both cases the potential energy decreases, however the particle moves in the opposite direction according to the sign of the charge.
Recall that the electric force was defined between two charged objects. The electric field, however, is used to expresses a property of a single charged object. The electric field was defined as follows:

\[ E \equiv \frac{F}{q} \]

So that once the electric field is known in some vicinity of space we can find the force on a charged particle brought into that space as:

\[ F = qE \]

The relationship between the electric potential, symbolized as \( V \), and the electrical potential energy is exactly analogous to the relationship between the electric field and the electric force. Therefore, we have.

\[ V \equiv \frac{U}{q} \]

\[ U = qV \]

The units of electric potential are \( J/C \), however, as this this quantity is so frequency used, it was given its own unit; (volt, V). It is named after Alessandro Volta, who is credited with creating the first electric battery.

Let’s look at a small example to illustrate the above concepts. Suppose we connect a 100 V battery to two conducting plates. We then place a proton near the positive plate. What speed will the proton attain when it reaches the negative plate?
We can start by directly using the conservation of mechanical energy, knowing that the initial kinetic energy is zero.

\[
U_i + K_i = U_f + K_f
\]

\[
K_f = -\Delta U
\]

\[
\frac{1}{2} mv^2 = -q\Delta V
\]

\[
v = \sqrt{-\frac{2q(V_f - V_i)}{m}}
\]

The proton ends at the negative terminal, which is at a lower potential. We can arbitrarily set this to voltage to \(V_f = 0\), which gives us \(V_i = 100\). Substituting we have the following:

\[
v = \sqrt{-\frac{2(1.6 \times 10^{-19})(0 - 100)}{1.67 \times 10^{-27}}}
\]

\[
v = 1.38 \times 10^5 \text{ m/s}
\]
**Final Summary for Electric Potential**

### Work and Electric Potential Energy
The work done by the conservative electric force from a charged object on a test charge, \( q \), is equal to the negative change in potential energy of the test charge – object system.

\[
W_C = -\Delta U = -(U_f - U_i)
\]

If the test charge starts at an infinite distance from the charged object where the potential energy is zero we can write:

\[
W_C = -U(r)
\]

Where, \( r \) is the distance between the charged object and the test charge.

### Electric Potential
The electric potential is defined as the electric potential energy per unit charge.

\[
V \equiv \frac{U}{q}
\]

Therefore, if a charged particle is placed at a point where the electric potential is \( V \), the potential energy can be found as follows:

\[
U = qV
\]

### Mechanical Energy
If a particle moves through space where the change in electric potential is \( \Delta V \) without any external forces acting, then the change in kinetic energy is given as:

\[
\Delta K = -q\Delta V
\]

And if the initial kinetic energy is zero, we can find the final speed of the object.

\[
K = -q\Delta V
\]

\[
\frac{1}{2}mv^2 = -q\Delta V
\]

\[
v = \sqrt{-\frac{2q\Delta V}{m}}
\]

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